**Mathematical instruction practices and classroom environment in China: a preface**

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Reform in China has some qualities in common with ‘reform’ in other countries, in that it aims to move teachers from a mainly transmission approach requiring learners to acquire knowledge and skills towards a combined approach to content, teaching and assessment which included application, problem-solving, independent thinking, and creativity. The curriculum and teaching methods should relate more closely to students’ interests and their ways of thinking, and exploration and practical activities should be used to give students some direct experiences. Change was initiated nationally with new standards and materials in 2001 and the nature of examination questions has also undergone change as the proportion of objective, procedural questions has fallen while the proportion of exploratory, subjective questions has risen. However, local examinations and accountability systems still depend to a great extent on pencil-and-paper traditional tests. In some countries, reform in mathematics teaching has followed shifts towards student-centred values, the broadening of the educational aims of school mathematics, and moves towards dialogic student-teacher relations. In China, by contrast, it has been said that curriculum reform is intended to lead changes of values, curriculum breadth, and classroom relations (Liu and Li, 2010). Nevertheless, as a European educator, I see many parallels. It can be both comforting and frustrating to read about the problems associated with curriculum and pedagogic change in various countries - comforting because similar issues about adaptation emerge everywhere, frustrating because one hopes for new insights about familiar issues. These four chapters taken together as a group offer some fresh perspectives, and I believe these to be perspectives that transcend the particular cultural context.

I am going to start with chapter 10 by Ding and Wong, in which relatively large samples of students and many teachers are asked about primary school classroom environments and teaching methods using various frameworks and analytical tools. A continuum from traditional to constructivist environment is assumed and teachers are found to be in the process of moving from one end to the other. Several indicators of change arise, such as increases in the use of real-life situations and discussion. While the majority of classrooms surveyed were more or less securely working in what has come to be known as a constructivist paradigm, a significant group were transitional, and a few were still very traditional.

I want to pick apart some of the terminology used in such studies, because it often confuses several issues. Firstly, ’constructivism’ was originally a theory of learning, and if it is true that students construct meaning for themselves, rather than accepting it pre-packaged from teachers, then they are constructing meaning whatever the teaching is like. In classrooms where recall of techniques and facts are important, students are likely to construct the understanding that mathematics is a collection of facts and methods. In classrooms that focus on problem-solving, discussion and groupwork, students are likely to construct the understanding that mathematics is a collective, interactive, way of solving problems. Both of these characterisations avoid the full nature of mathematics. Both approaches can leave students unable to recognise, anticipate, apply and appreciate the implications of underlying mathematical structures. For example, total dependence on algorithms to divide numbers can avoid meaning-based realisations such as ‘dividing by twice the number gives half the quotient’, and using only ad hoc situated methods to solve proportionality problems can avoid understanding the role of multipliers. In both kinds of classroom the pedagogy determines whether students are trapped in remembered methods, or trapped in ad hoc methods, rise beyond these traps. Good teachers draw out the key mathematical ideas, whatever the lesson type. In chapter 9, Zhao and Ma describe four teachers who are trying to do that, i.e. to enable students to understand key mathematical ideas, and develop fluent use, through the new classroom orthodoxy of realistic mathematics, discussion and working with others. The affective and social aspects of classrooms appear to be the main focus, and the teachers adapt in different degrees to these.

Another terminological issue is the phrase ‘problem-solving’, which in some cultures means ’interpreting word problems as sequences of operations to be performed’, but in others means ‘solving complex problems through a sequence of mathematical subgoals, making and testing conjectures, until a justifiable answer is obtained’. Depending on what meaning is used, one kind of teaching is more likely to be successful than another. Comparative studies in US and UK (e.g. Senk and Thompson 2003) show that students who had experience of open-ended, extended, collaborative problem-solving tasks were better at solving such problems and also at tackling unfamiliar questions than those who were ‘schooled’ in particular methods, but we would not expect that finding to hold up if the problems to be solved are merely worded versions of traditional formats. In any assessment of learning, those who have been ‘trained’ to act in particular ways are better at acting in those ways. Further, there is international ambiguity in mathematics education about the use of real contexts. We need to know whether the context merely provides images for mathematics (such as mixtures of coloured balls to represent ratio), or motivation (such as being about football or pop music), or whether the problem arises from the context, with a context-dependent solution, or arises from the context but gives access to a more general mathematical solution method. In these four cases, we would expect context to have different impacts on learning. It is not because I am from UK that I draw attention to these confusions, but because I believe that glossing over the different relations between context and learning mathematics can obscure our understanding of what is available for students to learn.

Chapters 9 and 10 show that teachers are reluctant to give up control and allow negotiation, and chapter 9 shows clearly why this might be the case. On the face of it, Zhao and Ma give us familiar portraits of teachers trying to make sense of new requirements while also remaining true to their views of the mathematical needs of their students and the examination system. But why would they give up control if they are the experts on mathematics in the classroom? I do not believe this question is adequately answered in our field internationally, because some of the reform language, taken to extremes, would leave students stirring their own pot of ignorance and having to rediscover key mathematical ideas for themselves. Any global reader with recognise the underlying tensions. If some Chinese teachers are concerned that fluency and skill will not develop within a reform curriculum, then they are not alone. A 21st century view might be that we do not need to be fluent in methods that can be done instantly by hand-held digital technology, but a more nuanced view might be that it is still useful to (for example) recognise multiples when they appear, to anticipate the outcome of certain algebraic manipulations, to adapt and understand stages in a multi-step process, and to know when to use which operation and how it might contribute to an answer. If ‘student-centred’ is interpreted to mean ‘student-directed’ it is hard to see how these concerns can be addressed.

The reason I have referred to the chapters in reverse order is because Chapters 7 and 8 give insights beyond these familiar issues and tensions and offer something distinctive, addressing the nature of subject content in lessons. This conforms with my own concerns that in focusing on the ‘how’ of mathematics teaching we can slip into ignoring the ‘what’. The key questions for me are: what is available to learn in mathematics lessons? what mathematical ideas and meanings can be constructed in this environment? I will start with the description of *bianshi* teaching by Wong, Lam and Chan. Variation theory is an ideal tool for thinking about planning teaching and textbooks, because it poses two related design challenges: ‘what are the key aspects of this mathematical idea?’ and ‘what dimensions of variation would give learners experience of these aspects?’ Nothing in the theory implies certain styles and structures of environments or even of lessons themselves. The theory confines itself to asking how the intended aspects of the subject matter are enacted in the lesson, and what lived experiences learners will have as a result. It focuses the mind on the available opportunities to learn.

The basic idea is that learners notice what varies against a background of invariance, or what is invariant against a background of variation. Learners then reason inductively from these experiences. Careful presentation of variation in tasks, whether they be textbook tasks or open-ended practical tasks, can reveal mathematical structure, methods, and can draw attention to relations, distinctions and therefore to concepts. This approach, championed in China by Gu and in the West by Marton and their colleagues (Gu, Huang & Marton 2004) does have something to offer and is beginning to be used not only to describe what is learnt but also to structure cognitive environments (e.g. Watson and Mason 2006).

Wong, Lam and Chan demonstrate how enactive and realistic examples provide varied entry points to a mathematical phenomenon, then varied iconic and diagrammatic representations provide access to a constant structure of relations, then varied symbolic representations describe a constant meaning. This meaning is then broadened by extending the range of change within a dimension of variation. Clearly, the lessons they describe are intended to be mathematically coherent; the examples and contexts give access to ideas that are gradually formalised and reified, then extended and built on further.

 Mok in chapter 8 approaches her analysis of teaching from the point of view of mathematical coherence rather than from overall lesson style. For coherence to be experienced by students, rather than merely intended by the teacher, procedural and conceptual links have to be created and developed through interactive strategies. To identify coherence requires analysis of how mathematical concepts develop within and across lessons, lesson routines, through classroom discourse – all of which are coordinated to make it as likely as possible that students will experience the intended mathematical ideas. I chose to read her analysis with a ‘bianshi’ lens in mind to see if there were parallels – and there are. Mok says: “conceptual links refer mostly to delineation and explanation of mathematical objects and skills; whereas, procedural links refers to application or expansion of skills and procedures” and, as is demonstrated in chapter 7, all of these can be approached through variation in tasks, examples and interactions.

Mok concludes that the coherence achieved by her study teacher depended on thematic connections, review, discourse, consolidation of variation, and summary. This reminded me how, in a study of lessons in three schools, we found that teachers with the weakest mathematical knowledge were least able to provide mathematically-focused consolidation, or to discuss mathematical implications of the work done, however good their generic teaching skills (Watson & De Geest 2011). For me, the most important feature of Mok’s chapter is that ‘what-why-how’ permeate the report of teaching. The ‘what’ can be achieved through talk associated with inductive reasoning from experience and examples; the ‘why’ by broadening the field of experience and building on other ideas; the ‘how’ by experiencing multiple representations and transformations, and the summary by generalising and reifying new conceptualisations. The reasoning required is more than learner induction. In chapter 9 it is clear that induction can lead either to imitation, or to a sharp focus on particular variation supported by talk. Clearly, this requires much more than either introducing groupwork or maintaining a transmissional approach to teaching and learning. Teachers need the mathematical knowledge to structure experience so that learners focus on key mathematical ideas.

The perspectives offered in these four chapters harness international concerns about changes in curricula and teaching methods, and turn our focus onto the way mathematical content is structured and presented for learners.

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