**Deep Progress in Mathematics:**

**making a difference**[[1]](#footnote-1)

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In the Improving Attainment in Mathematics Project, I and my colleague Els De Geest worked with ten mathematics teachers who were determined to make a difference to the attainment of those students who entered secondary school achieving below nationally-expected levels.

We wanted to tell the story of what they did and how they did it, but found out that there were very few common features in the way they taught – at least none which were obvious to observers. In other words, they used ten different ‘normal’ teaching styles, many different lesson structures, a wide range of task-types (from silent individual rapid written arithmetic to whole-class physical problem-solving) and at a superficial level all they seemed to have in common was the goodwill and energy required to make their teaching ‘work’. As researchers we were able to probe more deeply into what they did and what they said to us, and during the two years of the project we found that there were significant shared underlying principles to their work. These principles may be acted out in different ways, but that did not seem to matter so long as they were held and acted upon.

For example, one difference in how a principle was enacted was that all teachers felt that talk was important, so everyone should be engaged in classroom discussion, but for some teachers this meant that they repeated everything so that all students would *hear*, others would repeat nothing so that all students would learn to *listen*. All the teachers felt that personal organisation was important, but for some that meant students without anything to write with would have to scrounge from others or use inappropriate tools, others would give a pencil so that the students could get on with the maths[[2]](#footnote-2).

Our aim for the project was to find out what the teachers did and how they did it, but the shared aim *they* developed wasto influence students’ mathematical thinking so they become better learners. Between us all we invented the idea of ‘deep progress’ as a way to describe:

* Learning more mathematics
* Becoming better learners of mathematics
* Feeling better about learning mathematics

These three components can happen in any order, or all at once, or in any combination you like. Often an implicit focus on the last one is the best place to start with students who need rescuing, but the way to do that might be through focusing explicitly on the first; some teachers work the other way round, ensuring ‘fun’ and surreptitiously ensuring that some maths is learnt.

Of course we had long discussions about what ‘mathematical thinking’ meant, and tried not to be limited in what was expected, but at the very least all teachers expected students to think *hard* in maths lessons and stubbornly refused to simplify their expectations of how students would work, even if the content was fairly elementary.

Here is an example of the approach taken by one of the teachers. Sara started by asking her class to make four congruent squares, and gave them each a plastic wallet to keep them in. Then they had to find out ways of fitting them together, without overlaps, but with some kind of joins, and seeing what perimeters they could make:



You can imagine a lot of discussion as students have to sort out what the rules are, then begin to produce obvious ways to join the squares, fully edge-to-edge, and then need to record their findings somehow and see if they all have the same answers, but this doesn’t have to be done individually or in a particular way. Gradually, one of the students or the teacher might ask ‘is it possible to get a perimeter of 9, or one of 13…?’ and maybe someone will ask ‘can we have half edges joining?’ and so on as the exploration progresses. Classes, over maybe more than one lesson, may need notations for fractional joins, and might want to discuss how close you can get to 16 without the whole thing not really being joined except at the corners. Words like ‘perimeter’ and ‘vertex’ and ‘vertices’ are used regularly in the context of the task. There is a lot of talk and a lot of action but very little writing or drawing – indeed a teaching assistant can do the recording for the whole class. Finally the squares get put away and some questions asked:

* What do you know now that you didn’t know before?
* What can you do now that you didn’t know before?

Note that these are not closed factual ‘assessment’ questions about squares, but the answers *will* be factual and about squares, number, limits and so on.

Then a fifth square can be made or given and the whole task begins again – but it won’t be the same kind of activity at all because working with four squares in a sustained, discussed, way over time ensures that everyone now has more knowledge and skills to apply to the 5-square version. They can *feel* their own progress.

And so on ….

Finally they might have something worth writing about, worth communicating, worth drawing.

This kind of teaching is very complex and, repeated with other tasks over time, makes a real difference to learners:

* it focuses in personal organisation, and this is supported by the plastic wallets and the fact that they made their own ‘kit’ for the task.
* it triggers recall because their recent knowledge is used very soon after in a related task.
* it embeds the idea that simplifying can help you do more complex tasks.
* it needs thinking which goes beyond small integers and whole numbers.
* it recognises that learning involves shifts of focus, and it supports such shifts.
* it involves expressing ideas in shapes, words, numbers, maybe other symbols too.
* it is ‘accessed at their own level’ but everyone moves beyond their original level.
* the task goes on and on and on until everyone has had an intense experience of the changes it offers.

All this is rather different from the kind of teaching which limits low-attaining students to small whole numbers, arithmetical operations, neat low-level work, what ‘they can do’ easily, frequent changes of task because ‘they can’t concentrate’, no practical work because ‘they cannot look after equipment’, and no cognitive challenge because ‘they need everything to be step-by-step’. It has to be said, however, that one of Sara’s classes was very challenging and six months into the year, even though she has many years’ experience, she was still unhappy with their behaviour. Achieving success through such lessons is not easy, but all our teachers felt the struggle was worth it.

Sometimes classes can respond immediately to teachers who treat them as people who think. A student teacher who had been influenced by some of this work was asked to teach a low-attaining year 10 group, (S3 in Scotland), and decided to teach them how to solve simultaneous linear equations even though this topic was not in the syllabus for the examination they expected to take – and they knew it. The lesson I saw was last lesson of the afternoon the day before the end of term and there had been a fight at lunchtime, so not a propitious one for me to observe. I knew little of this. I did not know that it was a ‘bottom set’ and I only found this out by asking later. Rebecca started with a diagram like this, and asked them what value they thought a pink triangle would have:



With little difficulty they agreed it would be worth 4. The lesson progressed with several examples which were more and more complicated. Students discussed what values there would be, and there was a joyful combination of individual thinking, paired and whole-class discussion, as people suggested answers and reasons. Rebecca used the words ‘elimination’ and ‘substitution’ when appropriate but these were *post facto* as students devised these strategies for themselves. They were engaged and interested; some were literally on the edge of their seats. Half an hour into the lesson, against a distant hubbub from other classes, these students were still working hard. Finally they were working on situations like:



This lesson had many features similar to the 4-squares lessons. The teacher had engaged students visually rather than through text or symbols. There were high expectations that students would engage with hard thinking and social, visual and verbal support for them to do so. The progress towards complexity was managed through simple examples, but not a simplified view of what they were supposed to be learning. Written work was minimal, and technical terms were around to be used but did not get in the way of the main task.

You will want to be reassured that the project students did at least as well as comparison groups in standard test questions, and significantly better in questions which required some adaptation of original thought. In addition they were more willing and able to engage effortfully with non-routine work and extended explorations. Details of these findings can be found elsewhere (Watson and De Geest, 2003), but for this article it seemed more important to focus on the teaching, and to say clearly that there were no recipes or standard methods, just committed teachers trying things out and discussing them with each other.

By the end of the project, all teachers were more concerned with giving students time and space to learn than by covering the curriculum. They had learnt over two years that more can be achieved by improving learning. For example, one teacher spent about three weeks working with her year 7 class on the concept of ‘difference’ as subtraction and distance between. Visual and tactile approaches were used more and more, not as a ‘preferred learning styles’ approach but as a recognition that adolescents need multiple routes to access hard ideas, and mathematics provides them.

Most teachers gave students choices about how they worked, how to express their work, what order to do things in; they found that giving choice was more effective in engaging students than merely having the inevitable choice of whether to work or not. However, one teacher found that her students first had to be reminded of what it is like to work by being made to be silent and work on repetitive tasks lesson after lesson until they saw that they were beginning to make progress, get better, and learn more.

None of the teachers oversimplified the mathematical ideas. I have shown above two ways of leading students into complex ideas as extensions of accessible work.

Finally, although I have only given a brief view here of what we found, it is worth elaborating on the underlying beliefs which became established in the project and in the ways teachers worked:

We agreed that deep progress can only be achieved with all students if a teacher really believes:

* that *all* can learn mathematics
* that students ‘don’t’ rather than ‘can’t’ (e.g. ‘they *don’t* concentrate’ rather than ‘they *can’t* concentrate’) so the challenge is ‘how can I teach so that they *do*?’
* that focusing on mathematical thinking is an appropriate way to approach improvement
* that learners can change their goals in maths lessons from ‘to finish or to reject’ and ‘to fit in or to rebel’ to ‘to learn’
* changing habits is hard but achievable
* success in maths can be a source of self-esteem and empowerment

In addition there were beliefs about entitlement which we needed to state, not least because they can appear to be in conflict with many normal practices for low attaining students:

* all students have the right to, and are capable of, full engagement with the subject.
* all students are entitled to learn maths in ways which develop thinking and confidence in problem-solving.
* all students are entitled to have access to the maths necessary to function in society, beyond the minimal.

Of course, working together and sharing ideas and support was a critical feature for all the teachers.

**Bibliography**

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1. This work was developed during a research project funded by Esmee Fairbairn Foundation. The views expressed in all publications arising from the project are those of the author and associates. [↑](#footnote-ref-1)
2. Perhaps those Scottish teachers who attended my workshops in Stirling were a special group, but I noticed that overwhelmingly they claimed to be of the second type, whereas in England audiences tend to be more half-and-half about this difference. [↑](#footnote-ref-2)