IS THERE A GEOMETRIC IMPERATIVE?

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This article is an attempt to recreate some of the thoughts I tried to express in my lecture at the Higham conference. By preparing slides, posters, models, film and so on, but not preparing formal words, I had wanted to stress the point that geometrical experience lies in imagery rather than in the words that accompany or describe such imagery. To emphasise this I began with the Taoist motto: *the geometry that can be told is not geometry*. This does not contradict the useful classroom dictum – 'say what you see' – that John Mason had offered the previous day. But it indicated from the start my sense that geometrical experience is inherently unstable: as you become aware of it and articulate it, you will inevitably shift into some different experience, which I am tempted to call algebraic.

Algebra and geometry are really two sides of the same mathematical coin. I expressed this more suggestively at the time by quoting William Blake: *Time and Space are real beings; Time is a man and Space is a woman*. I interpret this to be saying something important about a fundamental duality in our lives; but this is not, for me, the duality of separate males and females. Broadly speaking, we might think of time as linear and ordered, perhaps as rational; but before we fall into too glib a correspondence, we might also think of time as stretching limitlessly into the past and the future, as eternal, infinite, abstract and uncertain. On the other hand, space might suggest something vast, oceanic, perhaps mystical or irrational; alternatively, we might think of space as enclosed, finite, concrete and safe. In so far as these yield a separation or division of our experience into two possibly opposing parts, then it becomes important to unite them. We have to live in one space-time; and we have to find both the masculine and the feminine in ourselves.

I also interpret Time and Space to be basic areas of mathematical experience. Time becomes manifest in the baby's slowly gathered sense of a before and an after and in the experience of rhythm and of repetition. According to Gattegno, you begin to be algebraic when, knowing how to say 'ah', you can then say 'ah-ah'. The medium for language, and so eventually for counting, is at first *sound*. Similarly, Space becomes manifest through the baby's slowly developed sense of body and of physical objects. Perceptions of the external world are internalised as mental images and these are the 'stuff' of geometry. Images derive from all the senses, but most typically and most powerfully through *sight*. Thus, Sound and Sight become yet another expression of the basic duality; and I associate these with algebra and geometry respectively.

Babies mobilise their powers of sight from a very early age. Their achievement in doing this so successfully means that they have already had some crucial geometric experiences. One way of framing a coherent geometry course round the few unrelated items offered in the National Curriculum, might be to think in terms of the geometric powers already available when infants start school. These may be considered in various ways. In my lecture, I grouped them under three headings:

imagining: seeing what is said

construing: seeing what is drawn
saying what is seen

figuring: drawing what is seen

These were illustrated in turn with various visual aids. The following account invokes the same material and is organised in the same way, but will inevitably be quite different.

Imagining

How can one describe adequately the miraculous way we translate words into images? In the case of the written word - say, 'the view of Skiddaw from the terrace at Higham' - light photons from the printed page are transformed into some mental activity: perhaps for some a vivid image of Skiddaw, perhaps for others another form of 'seeing', which might be described as a thought or a memory. In either case, what happens in the reader's mind will only very roughly correspond to what was in the writer's mind. But this process can become - magically - a communication.

The transformation of spoken words is even more miraculous in that they are so ephemeral. At Higham, I created a complex chain of communication starting from some words written by the novelist, William Golding, which I then read aloud. The passage I quoted came from the novel, *The inheritors*, in which Golding describes some dramatic events in the life of a group of Neanderthals. These remote ancestors of humans are conceived to have some language, but they are not all capable of retaining images or of associating them with events. Usually it is one older member of the group who can recall the falling of the leaves and the subsequent winter and so is able to recognise the signs that the summer season is ending, that it is time for the group to migrate.

A young male, called Lok, is only just beginning to recognise and retain images. He and the others are hungry. He tries to entertain everyone by boastfully dramatising his intention to get food. Another young male, Ha, is sceptical. The tension of their rivalry is broken by Fa, a young female. The extract began with Lok speaking:

> 'I shall bring back food in my arms' – he gestured hugely – 'so much food that I stagger – so!'
>
> Fa grinned at him. 'There is not as much food as that in the world.'
>
> (Lok continued:) 'Now I have a picture in my head. Lok is coming back to the fall. He runs along the side of the mountain. He carries a deer. A cat has killed the deer and sucked its blood. Under this left arm. And under this right one' - he held it out - 'the quarters of a cow.'
>
> He staggered up and down in front of the overhang under the load of meat. The people laughed with him, then at him. Only Ha sat silent, smiling a little until the people noticed him and looked from him to Lok. Lok blustered. 'That is a true picture.' Ha said nothing with his mouth but continued to smile. Then as they watched him, he moved both ears round, slowly and solemnly aiming them at Lok so that they said as clearly as if he had spoken: I hear you! Lok opened his mouth and his hair rose. He began to gibber wordlessly at the cynical ears and the half-smile. Fa interrupted them. 'Let be. Has has many pictures and few words. Lok has a mouthful of words and no pictures.' At that Ha shouted with laughter and wagged his feet at Lok ... Lok yearned suddenly for the mindless peace of their accord.

Golding has created a myth that expresses some truths about the mysterious link between images and words. It does not matter whether his imaginative reconstruction of Neanderthals is accurate, for we can never know how they thought or what images they had. Moreover, we can never really be sure we share the same images with our contemporaries, let alone people from the past. Yet, like Lok, we continue trying to do so and are often magically successful.

It could be fruitful to seek the nature of geometry within this magic. I emphasised this at Higham by having some prepared words of my own simultaneously read by someone else while also displayed with the overhead projector.

> Formal geometry is clearly magical, in the sense that its specialised language can adequately describe the real world. The French mathematician, Renee Thom, has also proposed the converse: *all magic, to the extent that it is successful, is geometry*. Such hermeneutics usefully shifts attention away from a search for the stuff of geometry in land measurements, stars and scales, to the mental processes involved in the creation and exchange of images. And it suggests a way of dignifying the teaching of geometry with a purpose that could be granted serious attention.

In a sense, words come easy. Like Lok we can babble away without owning much of what we say. The miracle is that words can trigger meaning and they do this by being associated with images of some sort - kinaesthetic, tactile, aural, as well as visual. This association is an internal private matter which no-one else can do for us. In his lecture at Higham, Dave Hewitt asked us to imagine leaving our house and going to the nearest shop. What do we notice when we take this journey in our imagination? What sights? What smells? Each of us owns our response in a way that no-one else can gainsay. It costs us very little to summon up such remembrances whatever form they take. We cannot not have some imagery.

I summarised the issue by another combination of written and spoken words, this time quoting Gattegno:

> Images are as common in our minds as words in our mouths. We live with them all day. They supply the bridge between the somatic awareness of our muscle tone and the fluidity of our thoughts, which seem to float in our heads linked with the verbal apparatus and our eyes... We live in our images and, in this sense, there is no reality that is not human.

Construing

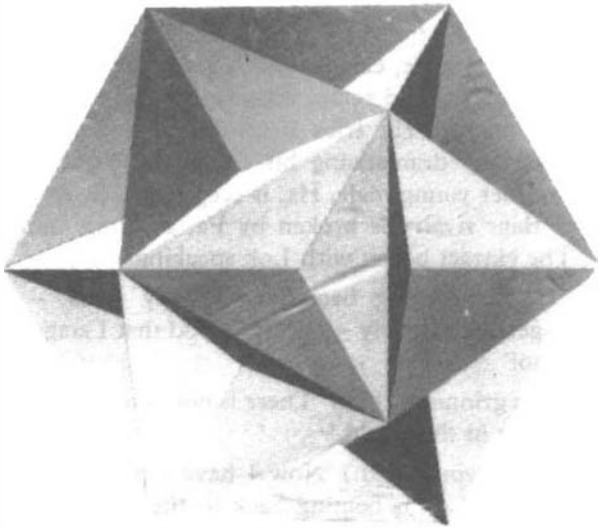
The pervasive influence of sight is nowhere more evident than in the use of the word 'sight' itself as a metaphor for understanding, as when we say we have insight, or when we 'see' something without necessarily having a visual image. In effect, all seeing involves a 'seeing', an interpreting of what our eyes are conveying to the brain. I gave four examples of such construing by showing various slides and a poster. Some of these are reproduced below with brief descriptions.

The first example was a picture of a horse and cart. An innocent eye might see the elliptical shape of the wheels, but the practised eye immediately construes the picture in depth and 'sees' the circular wheels. The implications of the whimsical possibility that the wheels are indeed elliptical are neatly described in Gelett Burgess' verse and some further drawings:

Remarkable
  Truly is Art!
See - Elliptical
  Wheels on a Cart!
It Looks very Fair
In the Picture, up There,
But Imagine the
  Ride, when you Start!

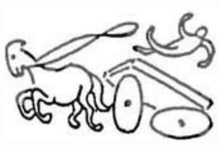
The need to construe two-dimensional pictures in depth was also illustrated by a selection of slides showing various lines and shapes in the environment. These were deliberately shown very rapidly to emphasise the further need to place a scene in some sort of context in order to construe it. A pattern of wavy lines might be a ploughed field, some tidal patterns in the sand, or a close-up of a rubber doormat. A sense of perspective and context is invoked in order to read pictures showing a reflection in a car bonnet, or some rotting fungus on a tree-trunk; but how can one know that a curious pattern of rectangles is found in the broken panes of a conservatory roof? Shadows provide the important clues for construing pictures showing a woman's face, or some steps. Such pictures may be used in classrooms to encourage people to construe for themselves and then to articulate what they see and to seek some shared description, perhaps – when appropriate – in standard terms.

The ability to see two-dimensional pictures in three-dimensional depth is really extremely remarkable. I do not know how and when it is achieved. But I have found it can be practised and improved; and people of all ages take a great delight in becoming more aware of this particular skill, one which is usually exercised automatically. At Higham, I had pinned up a poster on one of the side walls. This poster shows ... well, in one sense,



all it shows is a flat hexagonally bordered shape made out of various coloured triangles. Asked to describe it, most people speak in a way that suggests that they are seeing something three-dimensional. This is another example where it is useful to 'say what you see'. Hearing other people's descriptions can give you another point of view. Notice the visual metaphor – seeking and seeing other points of view is a basic mathematical activity. In this case, different ways of seeing can lead to at least four possible solids that the poster might be displaying.

It was only after I had worked with children on this poster for some time that a young nine-year-old once told me that it was a cube. It took me and the class some time to see it in this way. I was only using the poster at Higham to illustrate a point and did not intend giving time to letting people describe their own construing of the picture. So I drew it to their attention explicitly by asking them to look at 'the poster of a cube on the wall'. Some



people were able to construe the poster immediately in this way. Others were not helped by the imposed words and expressed puzzlement. I recall telling someone that I would be able to get her to see a cube if we gave time to work on the problem. This, of course, presupposes a willingness to admit the possibility that there is indeed another 'point of view'. Quite rightly, we usually trust our own perceptions, and are loath to admit that something outside our ken can be the case. Treating mathematics as a communal, shared activity is one way of helping us transcend our necessarily limited perspectives.

Figuring

This is the third and last of the headings invoked as examples of the mathematical aspects of sight. All mathematics might be considered as 'figuring', for figures are commonly numerals as well as drawings. In the geometrical sense, figuring legitimately covers a very wide area of any curriculum. My own associations are indicated in the following list.

FIGURING

fitting — seeing 2D in 3D
my shadows — drawing 3D in 2D
hand-shadows — drawing a cube
stick shadows — drawing cube-shadows
sundials — framing objects
navigation
vanishing points
fore-shortening
painting — perspective
architecture — projection
illusion
photography — projective geometry
geometries

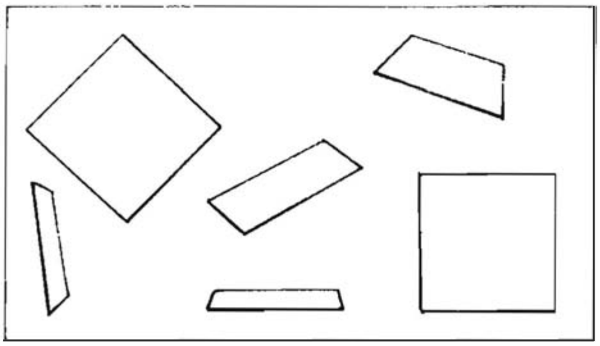
SPACE

Clearly only a few of these items could be illustrated in a lecture. I chose to offer a fairly rapid scanning of a few arbitrarily chosen developments from the problem of representing three dimensions in two; and I give a brief account of these items in this section.

It is one thing to 'read' a drawing of a cube in depth, quite another to draw something 'in perspective'. If young children have in a sense taught themselves the first skill, it seems that they have to learn the reverse from others in some way. It is always an instructive exercise to ask children at various ages to look at a cube and draw what they see. It is commonly found that many young infants draw a sort of topological picture of what they know to be a cube. This will often look like a flat net of six squares, corresponding to the six faces of the cube; sometimes there are only four or five squares. Many six- and seven-year-olds will continue to do this. But then, often it seems suddenly, they will draw the conventional drawing with the hexagonal outline and the three rhombic faces. I have often asked children how they first came across this: sometimes they say they learned it directly from parent or sibling, sometimes they recall copying someone else; rarely, it seems, is it directly taught in school.

In any case, the conventional drawing hardly ever portrays what is actually seen. This is also dramatically illustrated in drawings of a square. The conventional figure is rarely what people looking at an actual square would see. It represents, as it were, an equivalence class of possible perspectives of a square, including rectangles, parallelograms, rhombuses, and even line segments. So dominating, however, is the canonical representation that many young children will assert that a rotated version is a 'diamond' and emphatically not a square.

One useful way of calling attention to the infinite class of all perceptions – and corresponding representations – of a square is to project various views of a square on a screen; people are then asked to view these through a small square hole, cut out of card or paper, and to manipulate the card so that the shape on the screen is 'framed' exactly by the hole. (At Higham, I provided people with sheets of paper in which I had roughly cut out 1cm squares by folding the paper and cutting an oblong out of the folded edge; this was done before hearing David Fielker's lecture in which he mentioned the young boy who had done the thing more efficiently by cutting out a right-angled triangle!).

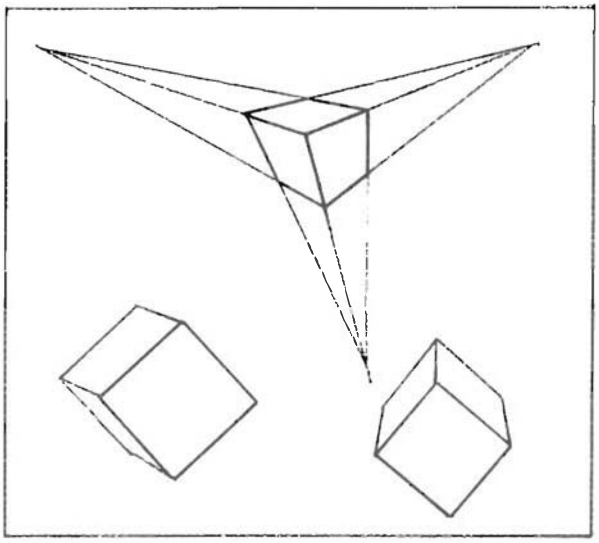


There are a number of useful – and amusing – variations on this activity. In the first place, someone can be asked to take hold of the screen and manipulate it so that the projected image is now intercepted by the screen as a standard square. Then, one might dispense with slides and merely ask someone to hold a square piece of card for the rest to frame in their holes. The different possible projections may now be achieved continuously, as the person holding the square card turns and twists it slowly in different ways. Finally, one may ask whether the projected image must always have opposite sides parallel in order to be framed in a square hole. For that matter, does the hole always have to be square? Would any four-sided hole frame any other four-sided shape? These are interesting geometric questions which can in the first place be investigated empirically.

The projections of a square are, in effect, possible representations of the square when viewed from various observation points. It is interesting to note that historically it was architects and painters who first explored such problems of representation, before mathematicians considered them. They did do so gradually, through a process of trial and error. Thus, in Western Europe, the rapid development of painting from the thirteenth century onwards, meant that painters assiduously explored ways of achieving depth in their pictures, and began to delight in accurate representation of buildings, receding scenes, chequered floors, and so on. So familiar are we now with the standard rules of perspective that the innocence of these rules in earlier painting can have a special charm for us.

As a reminder of the achievements of the Renaissance painters, I showed a collection of slides of pictures, mainly by Italian masters, working through these fairly rapidly in order to force people to think only in terms of the perspective problems being solved or not solved, as the case may be, and not to linger on detail or to seek contexts or meanings. This did little justice to artefacts whose interest and value lie in their emotional impact and the meaning that people have invested in them. But they serve to remind us of this potentially interesting example of a very important 'ethno-mathematical' exploration that was achieved long before the professional mathematicians took it up.

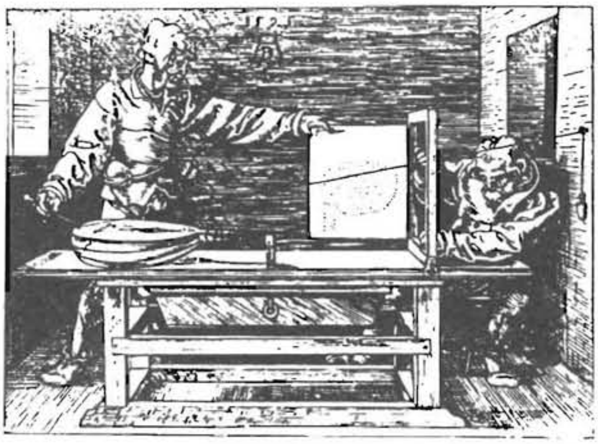
Some of the experiments conducted by the early investigators of perspective can be replicated fairly easily in the classroom. Thus, finding observation points by experiment can be surprisingly effective, for it uses intuitive skills in being able to 'read' pictures in depth and in being able to recognise when the perspective seems right. A representation of a cube with three random vanishing points seems at first to give a rather distorted view – like one of those Sienese paintings. But try shutting one eye and moving your head around until the cube suddenly 'clicks' into a natural seeming representation. For the drawings shown below, your eye should be closer to the page than reading distance. There is some intriguing geometry behind all this. For example, in the case of drawings with three vanishing points, it turns out that when the vanishing points form an acute-angled triangle, the observation point will be at a certain distance above the orthocentre.

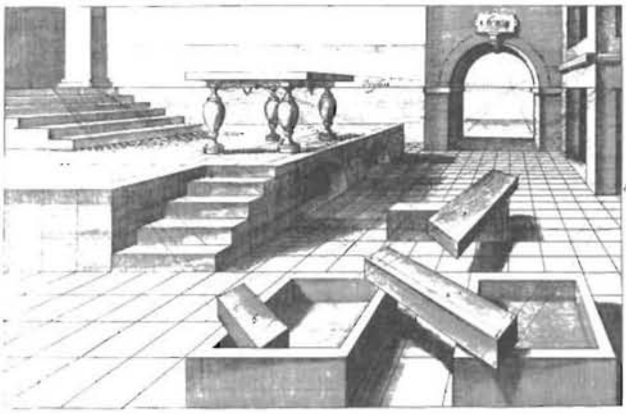


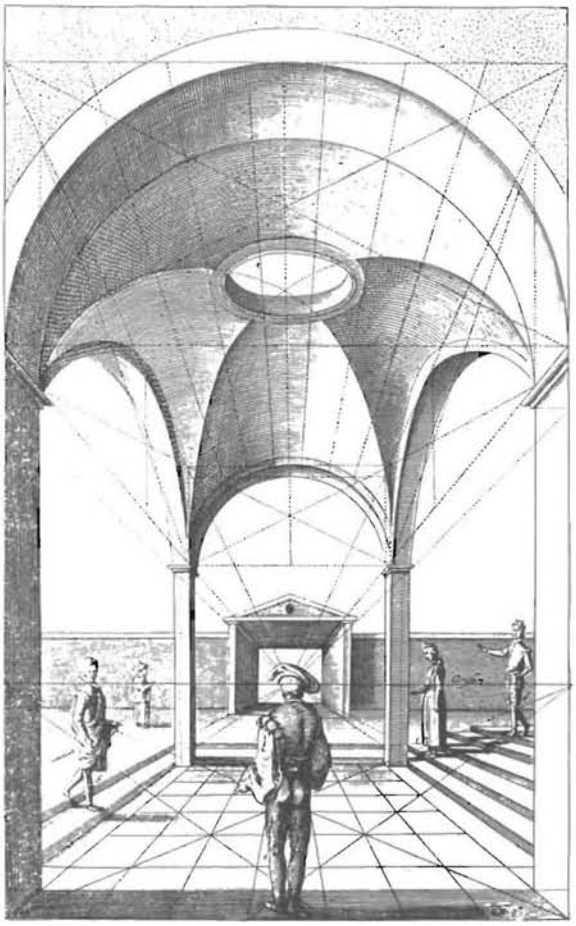
By the 17th century, a very detailed and sophisticated understanding of rules of perspective and projection was available to painters, architects, draughtsmen and engineers. Artists had been developing new ways of seeing for some time. The development of telescopes and microscopes stimulated new ways of seeing in science as well. Many of the Renaissance painters had been interested in mathematics. It seemed inevitable that someone would begin formal mathematisation of the practical rules of thumb for perspective drawing. The first to do so was a 17th century architect and engineer, Girard Desargues, though it is one of the ironies of history that his work was not widely taken up until the 19th century.

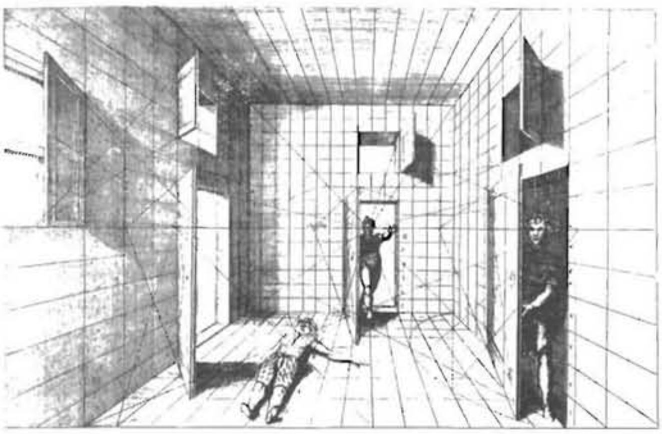
For Desargues, the problems of representing three dimensions in two were problems of projection; and he set out to analyse such transformations in general. One of the drawings on the opposite page shows the four lines of sight from a viewer's eye to the four vertices of a square. The ground yields a particular cross section of this pyramid. Other cross-sections would provide different projections. How are such cross-sections related? Consider triangular 'pyramids of sight'; for example, the rays of light from the sun to the corners of a triangular hole cut in the wall of a house. The sun casts a triangular patch of light on the floor inside. Each edge of the triangular hole and the corresponding edge of the patch on the floor will when produced meet on the skirting.

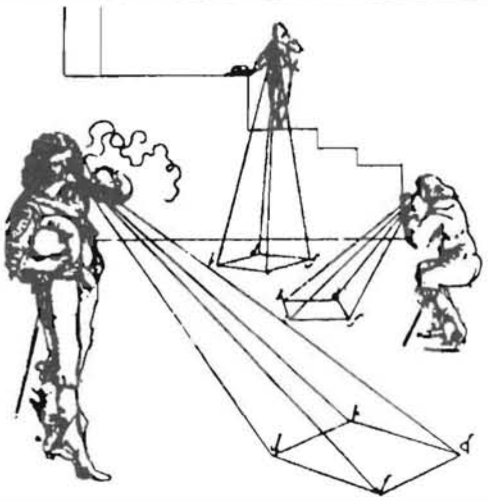
This is easier to understand when the image is talked through rather than written about. But in any case, the argument depends on being able to 'read' in depth the picture offered, or the mental



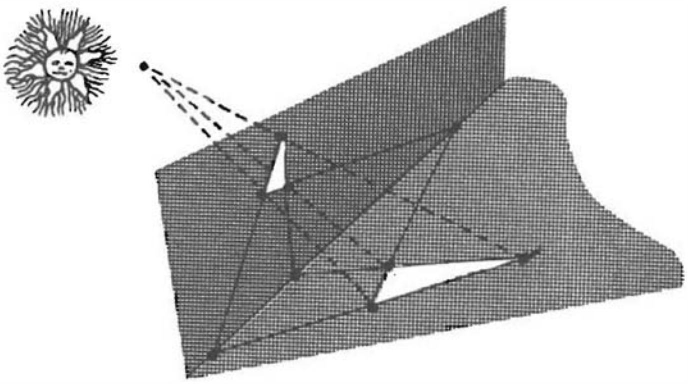








John Vredermann de Vries, *Perspective*, Dover Edition



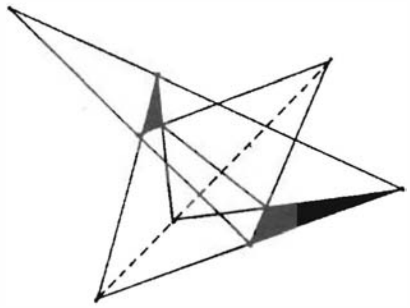


image invoked. At this stage, one may be asked to ignore the three-dimensional reading and asked to reconsider the pattern of lines as a two-dimensional diagram. When the picture is shown with an overhead projector, a deft switch of acetate sheets can remove the three-dimensional cover story about the sun shining through a hole in a wall. Now, you might see two coplanar triangles 'in perspective' from a point, and you might then assert that the three intersections of the three corresponding pairs of edges must lie in a line - the line that was previously along the skirting.

Whatever and wherever the triangles ... This is a powerfully general statement - a fundamental cornerstone of projective geometry known as Desargues' theorem. It creates a configuration of points and lines without specifying distances: ten points lying in threes on ten lines which meet in threes at the points. It is particularly suitable for the sort of exploration with a micro using the CABRI program that some people were working with at Higham. But it is also accessible through reflection on mental imagery, or through the mental construing of an actual picture. As such, it provides a useful example of the sort of geometrical thinking that falls within the stated geometrical demands of the National Curriculum.

There are lots of other interesting activities involving two-dimensional representation that can be designed for children of various ages. Though there are some good sources from which we can borrow suitable material, changing it for our own particular purposes, this has not yet been done in any systematic way. It would be a valuable service to others if a group of teachers undertook to try out various ideas in their classrooms, and then to prepare a coherent series of source books and, perhaps, workbooks for children, in the field of two-dimensional representations of three-dimensional space.

Points of view

The renaissance painters were preoccupied with the point of view – in the literal sense. Some of the most interesting paintings are those which by various means include the presence of the artist painting the picture, so making a link between the observation point and the vanishing points of the picture plane. When precise rules for determining perspective had been formulated, people felt free to explore other 'points of view'. Modern painters now deliberately flout the accepted rules of perspective in order to explore other ways of seeing, other perspectives. They have also taught us to look afresh at the paintings of other cultures and other times. We are now more likely to appreciate ancient Egyptian frescoes, Byzantine mosaics, Norman tapestries or Armenian miniatures, as well as the works of modern painters, in their own terms, rather than seeing them as failing in some way to represent reality properly.

Notions like 'perspective' and 'point of view' have now become general metaphors to be used in all sorts of contexts. But, perhaps surprisingly, this is not always so in the case of the word 'space' which still carries for many people the notion of some given reality, something absolute. The National Curriculum refers to 'shape and space', coyly adding 'geometry' in brackets. It does not thereafter say much about space; neatly begging various questions, it refers to 'the physical world'. But there is a lot of reference to 'shapes' and it is clear that these are conceived as being embedded in an unquestioned way, in a given – and unproblematic – space.

But, since the 19th century, mathematicians have developed their use of the word 'space' into a very general, and very powerful, notion. This is gradually becoming more familiar as a metaphor: people demand space, make space, give space, and are sometimes spaced out... My list of topics associated with figuring ended with the word 'space' and I see this as a general place-holder for what geometry might be about. It is the nature of the embedding space that determines 'shape'. Geometry must needs study space. But there are many spaces to be explored.

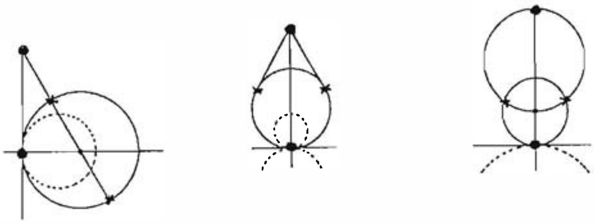
The reality of some seemingly strange spaces has often been emphasised by people who are in some way disabled. For example, one highly articulate woman suffering from Parkinson's disease told her doctor, Oliver Sacks, that when – in the grip of the disease – she was suddenly seemingly frozen into immobility in the act of going up stairs, what was happening to her was not as it might have appeared to an observer.

> It's not as simple as it looks. I don't just come to a halt. I am still going. But I have run out of space to move in. You see, my space, *our* space, is nothing like your space. Our space gets bigger and smaller. It bounces back on itself and it loops itself round till it runs into itself.

Another example comes from the observation of Helen Keller, who was blind and deaf from birth. With the help of a particularly sensitive teacher, she was able – it seems almost miraculously – to learn to speak. How could she understand such strongly visual notions as a straight line? She described it in terms of having to do something that could not be set aside: 'I feel as if I were going forward in a straight line, bound to arrive somewhere or go on for ever without swerving to the right or to the left.' This brilliant description of straightness owes nothing to visual experience. It reminds us that imagery does not have to be visual. It suggests an important characterisation of straightness in terms of a sort of symmetry, an *indifference* to alternatives. It also reminds us that it is very often 'normal' experience that is impoverished.

To accommodate these and other points of view, space has to become a symbol – and, according to Spengler, this is a religious symbol, representing 'the only method of actualising itself that the soul knows'.

Of course, such flights of fancy can easily become too far removed from some of the more mundane demands of the National Curriculum. In order to return to these, I had intended, at this point of my lecture, to introduce some more images by showing a brief silent animated geometry film, one of the new versions made by Gattegno of some of the films of Jean-Louis Nicolet. The film I had chosen was about... Well, it depends on your point of view! In one sense, it may be said to be about a certain cubic curve that happens to be called a *strophoid*. The locus is generated in three different ways; these are then shown simultaneously in a final sequence. So the film might be said to be about loci; on the other hand, the film involves families of circles and the film might be seen as illustrating some properties of circles. Or, taking a completely different point of view, it might be about taking different points of view! Having understood one way of looking at the curve, you have to see another way, and another, and another ... and then all of them simultaneously.



As it happened, I chose not to show the film during my lecture; a few people saw and worked on it the following evening. But I mention it here because it is a particularly effective vehicle for exploring points of view and because I wish to refer to it in the following final section of this article.

The geometric imperative

My final remarks will be as condensed as they necessarily were at the end of my lecture. They indicate *my* current answers to some of the questions being posed at the conference, including the one I asked in my title. I draw on the work of Caleb Gattegno for most of these.

Of course, there is a geometric imperative. One cannot not do geometry. Though how much one wishes to reflect on what one does is another matter. But if 'space' has indeed become 'an actualisation of the self', then there can be no question that there is more to space-exploration than sending rockets to the moon. Though even that can be seen from a psychoanalytic point of view – another perspective – as a phallic metaphor.

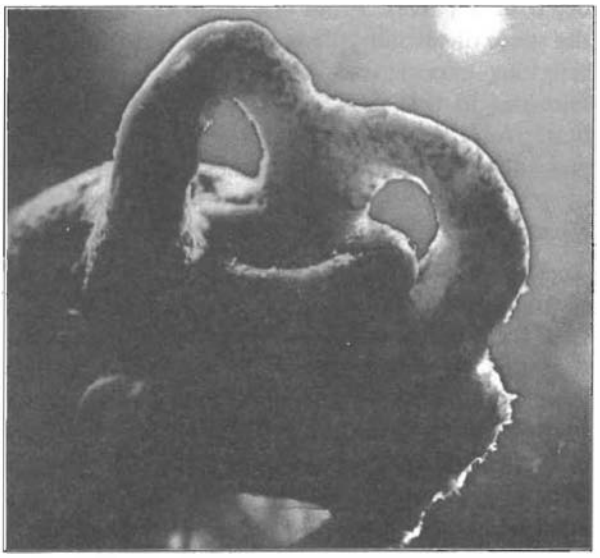
The first space we all inhabit is that of our mother's womb. This is indeed one sense in which space is a real being. Breaking out of that space involved separation, the crossing of a boundary. The duality of inside and outside becomes a symbol for all the other dividings that occur in our lives. From one point of view, this is the space of sexuality; from another it is the field of religious experience. At any rate, one way of knowing in this case is ecstasy. Though I note, tentatively, that the male in each of us can be so wary of ecstasy that he sometimes prefers to tame it by trying to bring it under rational control. The Neanderthal Lok likes to algebracise.

To be aware of these dimensions of space does not mean, of course (thank goodness, some might say), that they are to be explored explicitly in classrooms. The power of symbols is that they can do their work unseen. This can often be healing work: the word 'symbol' derives from a Greek root meaning 'putting together'. So when dealing with simple geometric elements such as lines and circles, which we know can have enormous unconscious meanings for people, it is as well

that we work at the conscious level.

If we are rightly cautious of speculation about the psychic origins of geometry, we can be more certain, and more explicit, about some somatic ones. The pictures on the opposite page dramatically capture a moment of important self-discovery, an actualisation of independence and self-control. Once we manage to control our limbs sufficiently to be able to get them to lever us up into a standing position, how is it that we don't fall back? Well, of course, we do in the first instance. But there are those marvellously delicate organs of balance inside each ear, the three semi-circular canals which enable us to keep upright, which give us a sense of balance, so – ultimately – our sense of symmetry. These three canals (shown in magnification below) are roughly perpendicular to each other: we carry three planes of symmetry around with us all the time. Here is an algebraic as well as a geometric imperative, for symmetry is, in a sense, not an intrinsic characteristic of space, but the expression of a certain sort of imposed relation.

Perhaps we can say that there is a natural - imperative – *awareness* of geometry, the awareness that enables us to stand upright, to take our bodies through doorways, to interpret the apparently diminishing figures of people walking away from us, and so on. But to become geometers we have to become aware of such awareness. And this is not an imperative, in the sense that not everyone will want to do this to the same extent.



Semicircular canals

But there are always certain aspects that will be peculiarly relevant. At different stages of our life, certain ways of knowing about ourselves and about the world are more important to us than others. For example, for young children the most important and accessible ways of knowing are through *perception* and *action*. Since all imagery is ultimately based on perception and action, and since geometry involves an awareness of imagery, it follows that young children may come to a *sense of truth* in geometry through sensing and acting. This is why we offer young children tiles, geoboards, blocks, card, and so on, to manipulate and reflect upon. Achieving and practising a personal, independent, authoritative sense of truth might be an appropriate reason for working geometrically with young children.

For older children, the situation is quite different. It is not good enough to offer adolescents merely more of the same experience they have had when they were younger. It is not good enough to use something as important and pervasive as geometry merely as a convenient medium for public examination and the selection of suitable candidates for higher education. Nor is it good enough always to *algebracise* geometrical experience, or to do so prematurely. What we do need to do, is to think in terms of the concerns of students themselves at this stage, and to see in what ways geometry can offer them something pertinent to their intellectual, emotional, social and spiritual needs. Of course, this is easier said than done. These are complex needs. In some ways, adults have to choose certain simplifications of them in order to survive, in order to be able to act in the world, however imperfectly. But it is the right of adolescents to explore *complexity*; and it is the duty of teachers to help them maintain it.

According to Gattegno, there is a way of knowing that respects and maintains complexity, which he calls *intuition*. In using intuition, we use the whole of the self; and this means bringing together the divided parts of ourselves whatever we conceive or name these to be. Perhaps I can now designate these parts as algebra and geometry. In our culture, it sometimes seems as if it is the geometrical part – we might call it right-brain thinking – which is under-used and under-valued. This means that there is an undue emphasis on

analytic, algebraic, left-brain thinking in our educational system. The geometrical imperative in secondary schools must surely be to mobilise neglected powers. This is, I think, best achieved by calling upon some of the ways of knowing that are specially relevant for adolescents. One such way of knowing is through affectivity. It is in geometry that adolescent students may exercise a sense of truth through feeling.

This is one reason why the neglected medium of animated film can be so powerful. Animation involves an infinite continuum of related images that generates an 'affective charge' which can enable viewers to recall some of the images and speak of them as they would of a familiar landscape. Such images may be 'owned' in a personal way and people can make statements about them with conviction. Shared reflection with others may change these convictions; but, in any case, communal discussion creates further possibilities that were not evident at first. In Gattegno's words, such films force a number of awarenesses, not least that 'I, too, can think of space creatively'.

But using film in this way requires a special discipline of its own, rather different from a passive viewing for entertainment. When a few people viewed the film about the strophoid one evening at Higham, some found it difficult at first to recall enough of the complex events that the film had enacted. This will be inevitable, of course, if we only have one point of view. But 'sharing' images allows a communal point of view. Allowing analytic anxiety to relax can permit release of intuition; this will include a 'knowing' through the feelings and through the senses, as well as through the intellect. Eric Love has rightly observed that after the terrible experiences of our century, we cannot afford to trust knowing through the guts too uncritically. So I emphasise that intuition uses all our faculties – and does so simultaneously.

The film portrayed a certain curve. Is there anything important about this curve? Nothing! It has some minor historical interest in that it was investigated in the 17th century by Isaac Barrow, and taken in textbooks as an example of a curve that could be defined either by a geometric construction or by an algebraic equation. Later, in the nineteenth century, it was arbitrarily named after the Greek word *strophos* (= twisted cord) by someone who fancifully saw in the curve the shape of a looped belt. This is hardly a vital chapter of geometry. But then nor is hardly any geometrical topic as such. Perspective, tiling, curves, cubes, squares, circles ... none of these matter as such. Each is an item in the *folklore* of mathematics. Any geometry syllabus is going to seem a more or less haphazard, traditional but idiosyncratic, collection of bits of folklore.

What *is* important is the awareness that there are geometrical awarenesses, or what John Mason, in his lecture at Higham, called the fact that there are facts. What is also important is the way the folklore is offered and addressed, the way in which people are invited to use themselves. With this in mind, I would sum up my sense of the geometric imperative by saying that geometry can – magically – invoke and develop powers of imagery, enhance our sense of truth, release and use affectivity, and elicit powers of intuition – enabling us to use the whole of ourselves.

The last words are with St Augustine who spoke about these matters so profoundly at the end of the 4th century AD.

> From words we can learn only words. Indeed we can learn only their sound and their noise. We learn nothing new when we know the words already, and when we do not know them we cannot say we have learned anything unless we also learn their meaning. And their meaning we learn not from hearing their sound when they are uttered, but from getting to know the things they signify ....
>
> In the halls of memory, we bear the images of things once perceived as memorials which we can contemplate mentally and can speak of with a good conscience and without lying. But these memorials belong to us privately. If anyone hears me speak of them, provided he has seen them himself, he does not learn from my words, but recognises the truth of what I say by the images which he has in his own memory. But if he has not had these sensations, obviously he believes my words rather than learns from them. When we have to do with the things which behold with the mind .... we speak of things which we look upon directly in the inner light of truth. ■