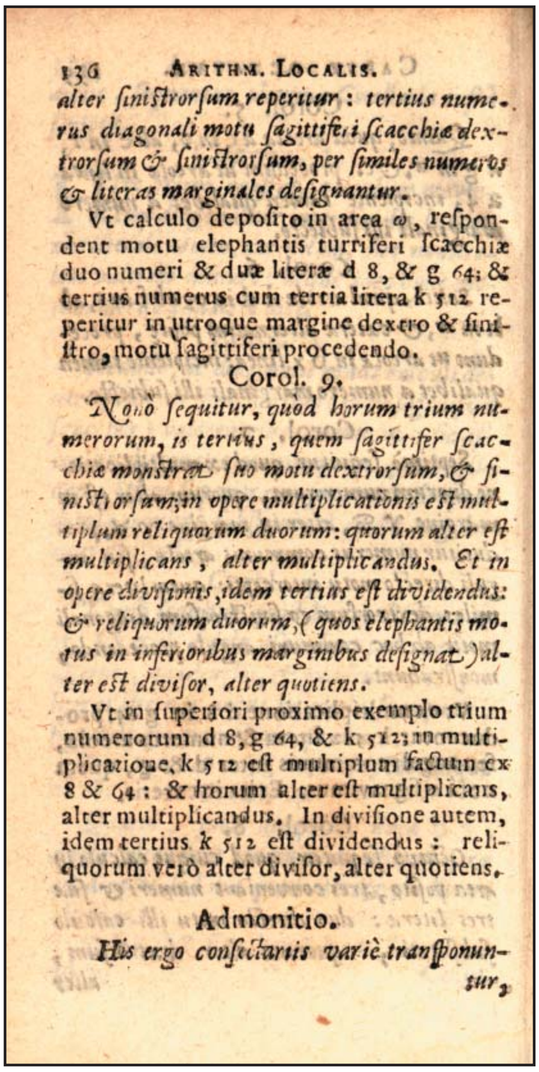


136 ARITHM. LOCALIS.

alter ſiniſtrorſum reperitur : tertius numerus diagonali motu ſagittiferi ſcacchiæ dextrorſum & ſiniſtrorſum, per ſimiles numeros & literas marginales deſignantur.

Vt calculo depoſito in area ω, reſpondent motu elephantis turriferi ſcacchiæ duo numeri & duæ literæ d 8, & g 64; & tertius numerus cum tertia litera k 512 reperitur in utroque margine dextro & ſiniſtro, motu ſagittiferi procedendo.

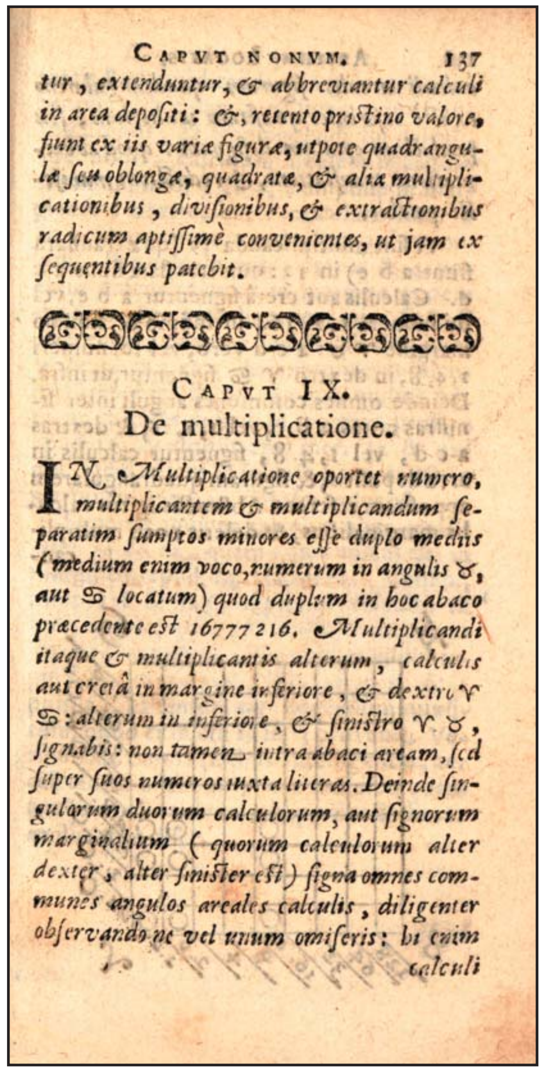
Corol. 9.

Nonò ſequitur, quòd horum trium numerorum, is tertius, quem ſagittifer ſcacchiæ monſtrat ſuo motu dextrorſum, & ſiniſtrorſum; in opere multiplicationis eſt multiplum reliquorum duorum: quorum alter eſt multiplicans, alter multiplicandus. Et in opere diviſionis, idem tertius eſt dividendus: & reliquorum duorum, (quos elephantis motus in inferioribus marginibus deſignat) alter eſt diviſor, alter quotiens.

Vt in ſuperiori proximo exemplo trium numerorum d 8, g 64, & k 512; in multiplicatione, k 512 eſt multiplum factum ex 8 & 64 : & horum alter eſt multiplicans, alter multiplicandus. In diviſione autem, idem tertius k 512 eſt dividendus : reliquorum verò alter diviſor, alter quotiens.

Admonitio.

His ergo conſectariis variè tranſponuntur,



CAPVT NONVM. 137

tur, extenduntur, & abbreviantur calculi in area depoſiti : &, retento priſtino valore, fiunt ex iis variæ figuræ, utpote quadrangulæ ſeu oblongæ, quadratæ, & aliæ multiplicationibus, diviſionibus, & extractionibus radicum aptiſſimè convenientes, ut jam ex ſequentibus patebit.

CAPVT IX.

De multiplicatione.

IN *Multiplicatione oportet numero, multiplicantem & multiplicandum ſeparatim ſumptos minores eſſe duplo mediis (medium enim voco, numerum in angulis ♉, aut ♋ locatum) quod duplum in hoc abaco præcedente eſt 16777216. Multiplicandi itaque & multiplicantis alterum, calculis aut cretâ in margine inferiore, & dextro ♈ ♋ : alterum in inferiore, & ſiniſtro ♈ ♉, ſignabis: non tamen intra abaci aream, ſed ſuper ſuos numeros iuxta literas. Deinde ſingulorum duorum calculorum, aut ſignorum marginalium (quorum calculorum alter dexter, alter ſiniſter eſt) ſigna omnes communes angulos areales calculis, diligenter obſervando ne vel unum omiſeris: hi enim calculi*

Chapter IX: Multiplication.

Make sure that both of your numbers can be represented by counters being placed lower than half way up the board (i.e., between ♈ to ♉ on one side and ♈ to ♋ on the other). On the board shown in his diagram each number must be less than or equal to 16,777,216.

Place counters (or otherwise mark) the positions in the margins for each number. A counter is to be placed at the intersections of each of the direct rows and columns of the ones you marked.

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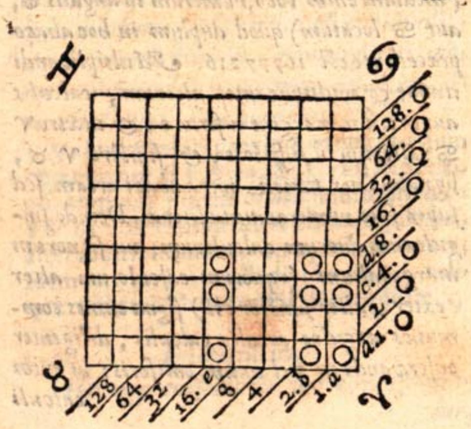
calculi areales figuram quadrangularem exactè referentes quæſitum multiplum, ſeu productum optatum deſignant: quod abbreviatione, tranſlatione, & reductione manifeſtè patebit.

Vt ſint multiplicanda 19 (quæ tranſlata ſiunt a b e) in 13: quæ tranſlata ſiunt a c d. Calculis aut cretâ ſignentur a b e, vel ſui numeri 1, 2, 16, in infimo & ſiniſtro margine ♈ ♉: a c d verò, vel ſui numeri 1, 4, 8, in dextro ♈ ♋ ſignentur, ut infrà. Deinde omnes communes anguli inter ſiniſtras notas a b e, vel 1, 2, 16, & dextras a c d, vel 1, 4, 8, ſignentur calculis in area depoſitis, & figuram quadrangularem appoſitam referent. Abſtractis igitur calculis marginalibus, & deletis notis multiplican-



cantis & multiplicandi, quæ prius apponebantur: abbrevianda eſt ſumma quadranguli arealis, & tranſponendi ſui calculi, hoc modo: calculum arealem in angulo communi inter 1 & 1, tranſpone in marginem 1 dextrorſum: calculum inter 2 & 2, in numerum marginalem 2 dextrorſum. Pro calculo etiam inter 4 & 4, ponatur unicus calculus in margine eodem apud 4. Pro calculis autem inter 8 & 8 auferendis, ponatur unicus calculus inter 16 & 16 in area: & jam ſunt tres calculi in area inter 16 & 16: pro quibus ponatur calculus unicus in margine præfato apud 16, & alius in area inter 32 & 32: qui, quia unicus & ſolus in hac area eſt, in marginem apud 32 transferendus eſt.

Supereſt inſuper calculus alius arealis inter 64 & 64, quem (quia unicus eſt) in marginem apud 64 transfero. Vltimò, inter 128 & 128, reperitur calculus in area, quem (quia unicus eſt) in marginem juxta 128 transfero. Et ita ex calculis marginalibus iuxta 128, 64, 32, 16, 4, 2, & 1 poſitis, habentur 247 multiplum quæſitum, quod ex ductu 19 in 13 provenit. Verùm, hæc omnia faciliùs intelliguntur, per calculos in abaco ampliore mobiles, quàm per hos in hoc alveolo impreſſos & fixos: ex illis ergo diſce.

Aliud

Napier provides the example of 19 times 13. The first number (19) is marked out on the margin of the abacus (in this case he notes this by the letters a,b,e on the ♈ to ♉ margin) and the second (13) by the letters a,c,d on the ♈ to ♋ margin. Counters are then placed on the intersecting squares. One must now abbreviate the total shown on the abacus (get it into its canonical form). This is accomplished by moving counters as follows:

The single counter found on the intersection of *a* and *a* is moved to the margin (see the counters at the extreme right hand side of the above diagram).

Similarly the single counter found between the margins labeled “2” is moved over, and the single counter between the “4” marks is moved as well.

There are two counters found between the points marked “8” and these are removed and one counter placed on the board in an empty 16 position—there are now three counters in the “16” position.

Two of the counters found at the “16” position are removed and one placed on and empty square in a “32” position while the remaining one is moved to the margin in the “16” position.

Move the single counters in the “32,” “64,” and “128” positions to the margin.

The margin will now contain counters in the 1, 2, 4, 16, 32, 64, and 128 locations which yields the product of 19 times 13 = 247.

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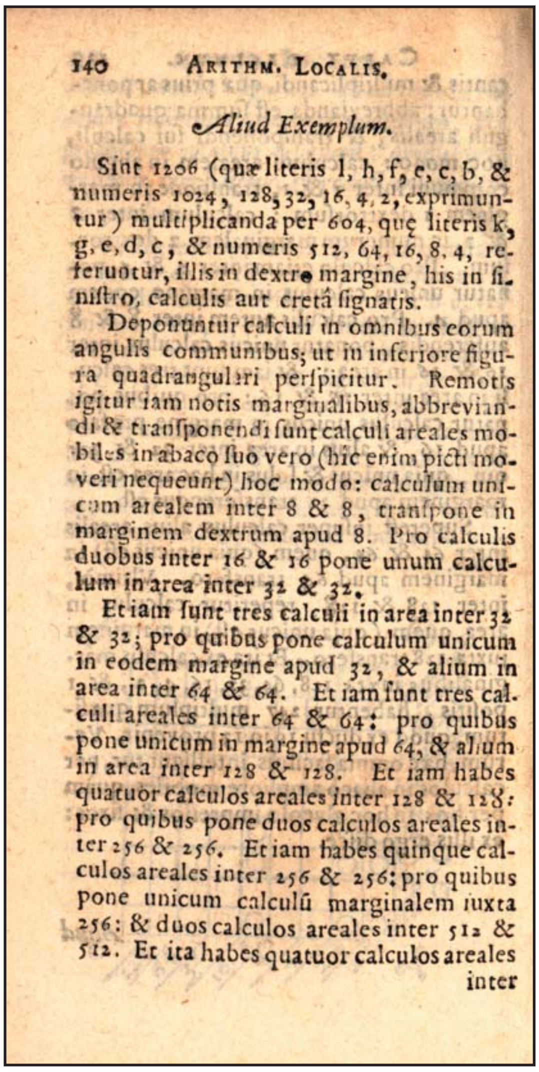


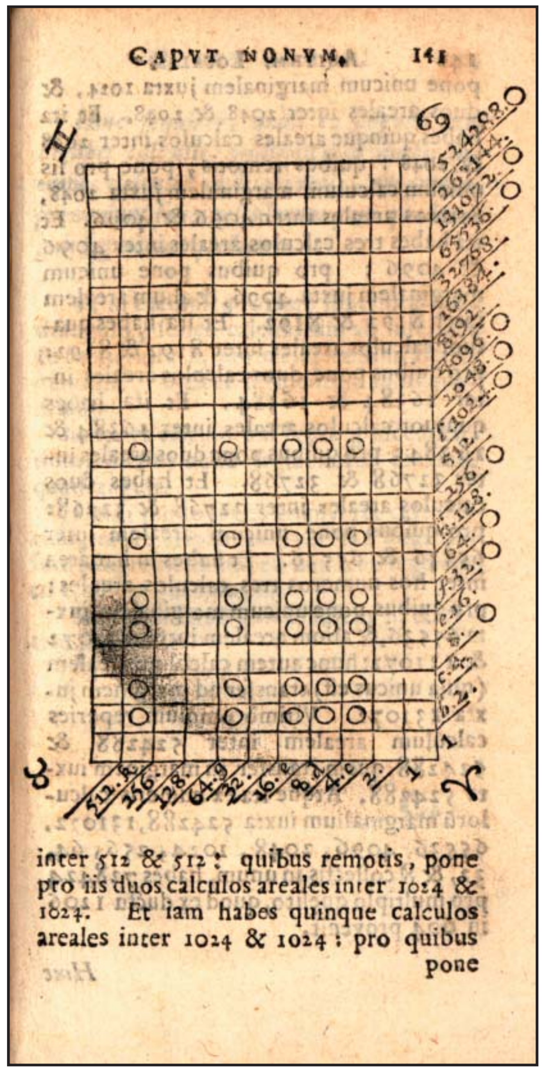
140 ARITHM. LOCALIS.

Aliud Exemplum.

Sint 1206 (quæ literis l, h, f, e, c, b, & numeris 1024, 128, 32, 16, 4, 2, exprimuntur) multiplicanda per 604, quę literis k, g, e, d, c, & numeris 512, 64, 16, 8, 4, referuntur, illis in dextro margine, his in ſiniſtro, calculis aut cretâ ſignatis.

Deponuntur calculi in omnibus eorum angulis communibus; ut in inferiore figura quadrangulari perſpicitur. Remotis igitur iam notis marginalibus, abbreviandi & tranſponendi ſunt calculi areales mobiles in abaco ſuo vero (hic enim picti moveri nequeunt) hoc modo: calculum unicum arealem inter 8 & 8, tranſpone in marginem dextrum apud 8. Pro calculis duobus inter 16 & 16 pone unum calculum in area inter 32 & 32.

Et iam ſunt tres calculi in area inter 32 & 32; pro quibus pone calculum unicum in eodem margine apud 32, & alium in area inter 64 & 64. Et iam ſunt tres calculi areales inter 64 & 64: pro quibus pone unicum in margine apud 64, & alium in area inter 128 & 128. Et iam habes quatuor calculos areales inter 128 & 128: pro quibus pone duos calculos areales inter 256 & 256. Et iam habes quinque calculos areales inter 256 & 256: pro quibus pone unicum calculũ marginalem iuxta 256: & duos calculos areales inter 512 & 512. Et ita habes quatuor calculos areales inter



CAPUT NONUM. 141

inter 512 & 512: quibus remotis, pone pro iis duos calculos areales inter 1024 & 1024. Et iam habes quinque calculos areales inter 1024 & 1024: pro quibus pone

Here Napier provides a second example of multiplying 1,206 times 604 (= 728,424).

pone unicum marginalem juxta 1024, & duos areales inter 2048 & 2048. Et ita habes quinque areales calculos inter 2048 & 2048: quibus remotis, pone pro iis unicum calculum marginalem juxta 2048, & duos areales inter 4096 & 4096. Et ita habes tres calculos areales inter 4096 & 4096: pro quibus pone unicum marginalem juxta 4096, & alium arealem inter 8192 & 8192. Et ita habes quatuor calculos areales inter 8192 & 8192: pro quibus pone duos calculos areales inter 16384 & 16384. Et ita habes quatuor calculos areales inter 16384 & 16384: pro quibus pone duos areales inter 32768 & 32768. Et habes duos calculos areales inter 32768 & 32768: pro quibus pone unicum arealem inter 65536 & 65536. Et habes in hac area inter hos numeros tres calculos areales: pro quibus pone unicum marginalem juxta 65536, & alium arealem inter 131072 & 131072: hunc autem calculum arealem (quia unicus est) transfer ad marginem juxta 131072. Vltimò omnium reperies calculum arealem inter 524288 & 524288, quem transfer in marginem iuxta 524288. Atque ita ex numeris calculorũ marginalium iuxta 524288, 131072, 65536, 4096, 2048, 1024, 256, 64, 32, & 8 collectis in unum, habes 728424 pro multiplo quęsito, quod ex ductu 1206 in 604 provenit.

Hinc

Hinc sequitur, quòd ex singulis quibuslibet calculis multiplicandi ductis in omnes calculos multiplicantis, aut contrà, proveniunt series calculorum quas quadranguli segmenta appellamus.

Vt in exempli proximè superioris quadrangulo, series calculorum ab inferiore & sinisteriore k, motu elephantino ascendentium, dicitur segmentum illius quadranguli.

Sic series calculorum supra g ascendentium, dicitur aliud segmentum eiusdem quadranguli.

Simili modo series transversa calculorum, motu elephantino versus l dextrorsum progredientium, est unum ex segmentis ejusdem quadranguli.

Sic etiam series quæ tendit in h, & cęterę similes.

CAP.

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CAPVT X.

De Diviſione.

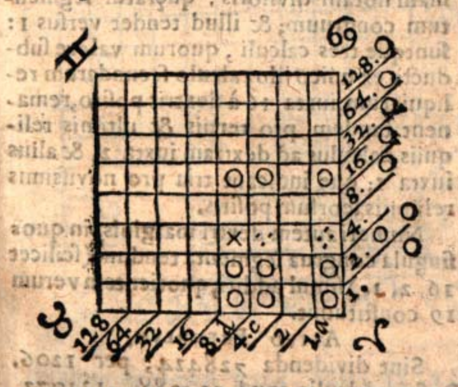
IN Diviſione ſagittifer à maximo calculo dividendi motu æquali, & elephas à maximo diviſoris monſtrant communem angulum, à quo ſeries calculorum diviſori undique parallela procedens, dicitur ſegmentum: congruum, ſi minus fuerit dividendo relicto: alioquin proximè illi ſubſtituta ſeries pro ſegmento congruo capiatur.

Vt mox per exempla in diviſione patebit.

Diviſio ergo ſic ſe habet: Numerum dividendum calculis in alterutro margine ſignabis, diviſorem autem (diſtinctionis gratiâ) notis in eodem, ſive in alio margine ſignabis. Inde horum ſegmentum congruum in area conſtitue: quo, vel cujus valore, ex dividendo ablato, obſerva calculos relictos: à quibus etiam dempto ſuo ſegmento congruo, notentur & hæ reliquiæ: à quibus iterum, atque iterum, auferantur ſucceſſivè ſua ſegmenta congrua: donec tandem aut nihil relinquatur, aut ſaltem numerus diviſore minor,

nor; & hic ſeorſim poſitus indicat noviſſimas reliquias. Numeri autem laterales alterius marginis, in quos ſingula congrua ſegmenta tendunt, ſimul additi, quotientem verum tibi referent.

Vt ſint partienda 250 per 13. Poſitis calculis in dextro margine iuxta 128, 64, 32, 16, 8, & 2 numeros, ſignetur 250 dividendus: poſitis autem in altero margine inferiore & ſiniſtro notis apud 8, 4, & 1, ſignetur 13 diviſor. Horum quære primum ſegmentum congruum hoc modo: Aſcende ab 8 infimo per motum elephantis, & progredere ab 128 ad dextram poſito per motum ſagittiferi: & à



communi utriuſque angulo, colloca ſeriem calculorum diviſori parallelam: hæc in 16 dextrorſum tendit, & eſt ſegmentum primum

G

Chapter X: Division.

Napier's division example (250/13) is shown in the diagram on page 145. The dividend (250) is set up on the ♈ to ♋ margin and the divisor (13) on the ♈to ♉ margin. Beginning with the largest position of the divisor (8) move to directly adjacent squares (in the diagram as shown it is "up") until arriving at the diagonal line denoted by the largest position of the dividend (in this case 128). Place counters on every position from the square thus found (square 8,16 in this case) that correspond to places in the divisor (thus counters go on the 8,16; 4,16; and 1,16 squares.

Subtract the value thus obtained (16*8 + 16*4 + 16 * 1 = 208) from the dividend leaving 42 (which results in marginal counters in positions 32, 8, and 2).

Similar to the first operation, take the largest value in the dividend (8) and move "up" until hitting the diagonal row containing the largest position in the remainder found in the last subtraction (32). Put counters on each square in the row from this location (marked with and "x") that correspond to each position in the divisor (that results in counters in the squares marked "x", "three dots" and "four dots"). The value of this number (52) is greater than the remainder found above (42) so move these counters down one row (from the row marked with an "x" to the row immediately below) and try to subtract this new number (26) from the original remainder (42) which now gives a new remainder of 42 - 26 = 16.

Repeat these operations until the quotient is found by noting the values being "pointed to" by the rows of remaining counters in the center of the abacus (16, 2, 1) and the remainder, if any, will be on the right hand margin.

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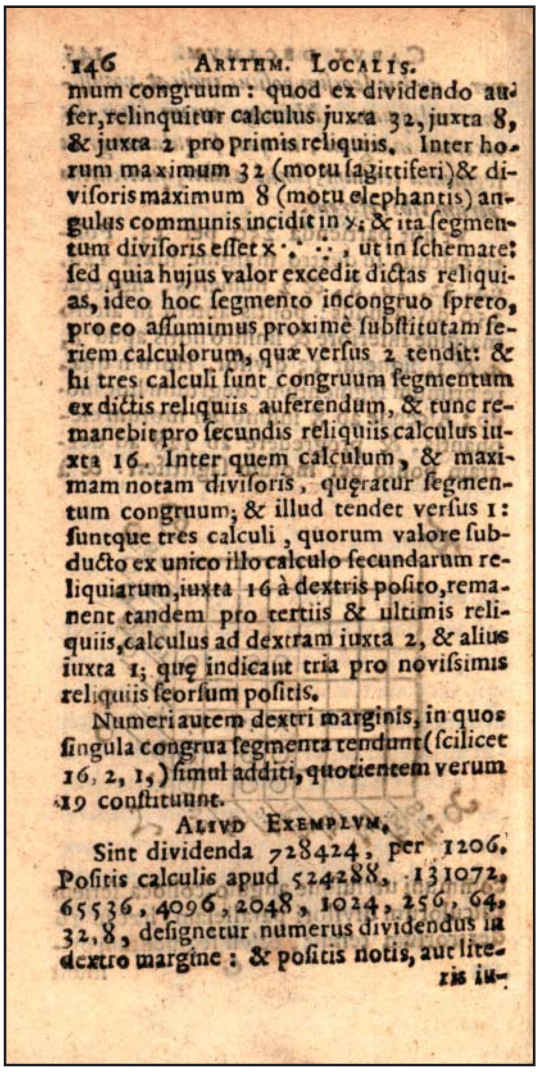


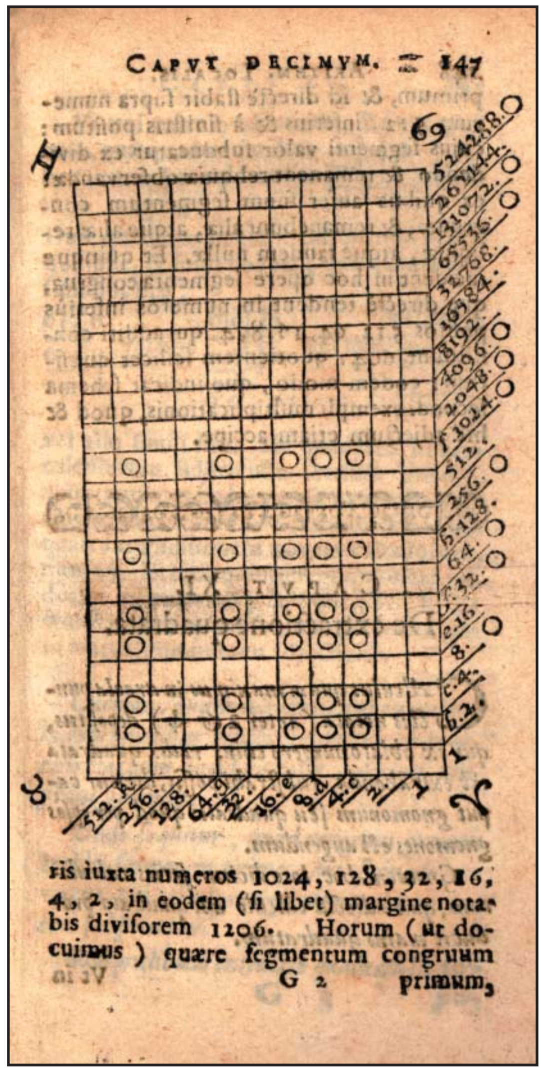
mum congruum: quod ex dividendo aufer, relinquitur calculus juxta 32, juxta 8, & juxta 2 pro primis reliquiis. Inter horum maximum 32 (motu sagittiferi) & divisoris maximum 8 (motu elephantis) angulus communis incidit in x; & ita segmentum divisoris esset x ·.˙ ·:· , ut in schemate: sed quia hujus valor excedit dictas reliquias, ideo hoc segmento incongruo spreto, pro eo assumimus proximè substitutam seriem calculorum, quæ versus 2 tendit: & hi tres calculi sunt congruum segmentum ex dictis reliquiis auferendum, & tunc remanebit pro secundis reliquiis calculus iuxta 16. Inter quem calculum, & maximam notam divisoris, quæratur segmentum congruum; & illud tendet versus 1: suntque tres calculi, quorum valore subducto ex unico illo calculo secundarum reliquiarum, iuxta 16 à dextris posito, remanent tandem pro tertiis & ultimis reliquiis, calculus ad dextram iuxta 2, & alius iuxta 1; quę indicant tria pro novissimis reliquiis seorsum positis.

Numeri autem dextri marginis, in quos singula congrua segmenta tendunt (scilicet 16, 2, 1,) simul additi, quotientem verum 19 constituunt.

ALIUD EXEMPLUM.

Sint dividenda 728424, per 1206. Positis calculis apud 524288, 131072, 65536, 4096, 2048, 1024, 256, 64, 32, 8, designetur numerus dividendus in dextro margine: & positis notis, aut lite-

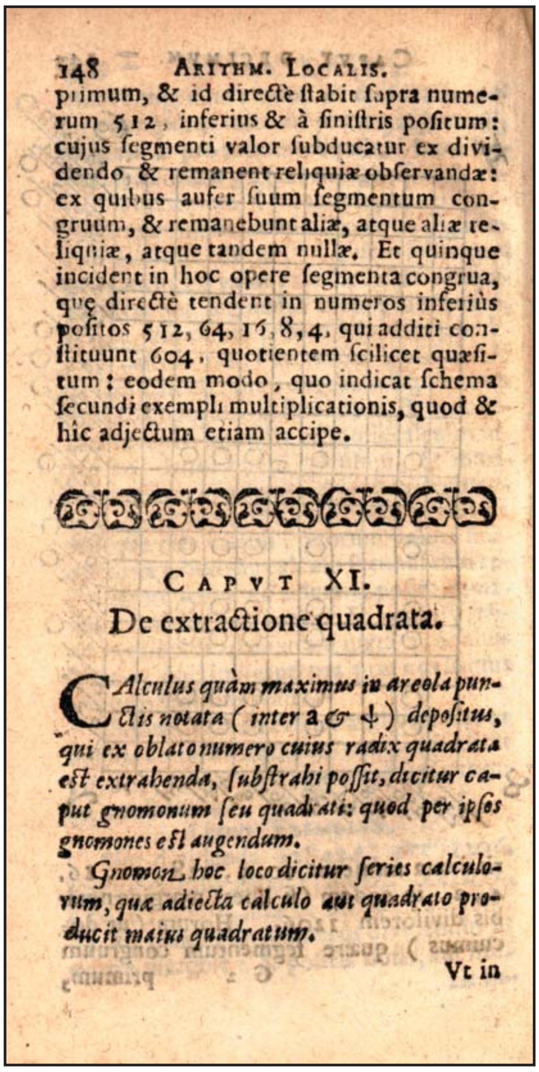
ris iu-



ris iuxta numeros 1024, 128, 32, 16, 4, 2, in eodem (si libet) margine notabis divisorem 1206. Horum (ut docuimus) quære segmentum congruum

 primum,

A second example shows the division 728,424/1206 = 604.



148 ARITHM. LOCALIS.

primum, & id directè ſtabit ſupra numerum 512, inferius & à ſiniſtris poſitum: cujus ſegmenti valor ſubducatur ex dividendo & remanent reliquiæ obſervandæ: ex quibus aufer ſuum ſegmentum congruum, & remanebunt aliæ, atque aliæ reliquiæ, atque tandem nullæ. Et quinque incident in hoc opere ſegmenta congrua, quę directè tendent in numeros inferiùs poſitos 512, 64, 16, 8, 4, qui additi conſtituunt 604, quotientem ſcilicet quæſitum: eodem modo, quo indicat ſchema ſecundi exempli multiplicationis, quod & hîc adjectum etiam accipe.

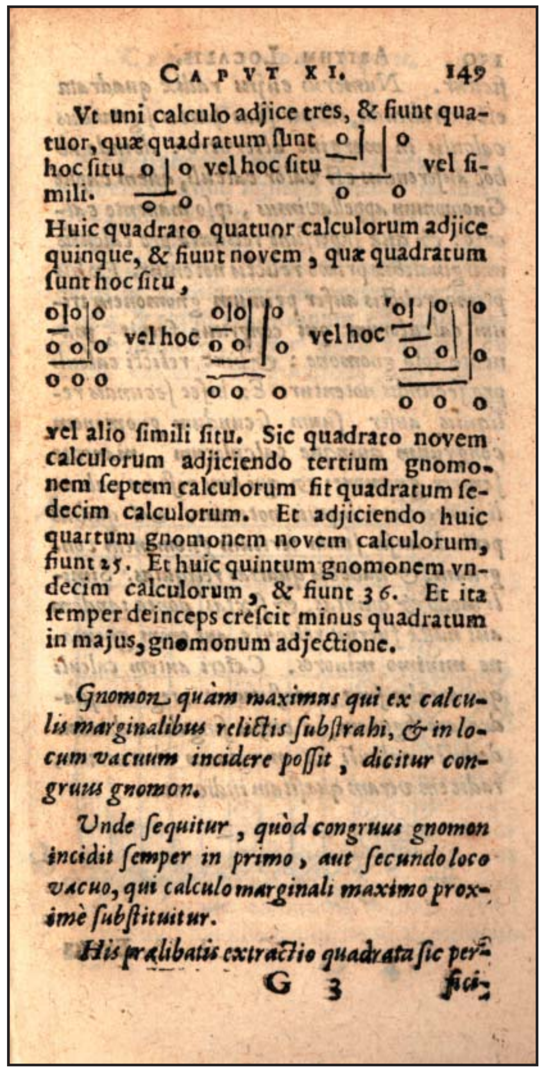
CAPVT XI.

De extractione quadrata.

Calculus quàm maximus in areola punctis notata (inter a *&* ✝*) depoſitus, qui ex oblato numero cuius radix quadrata eſt extrahenda, ſubſtrahi poſſit, dicitur caput gnomonum ſeu quadrati: quod per ipſos gnomones eſt augendum.*

Gnomon hoc loco dicitur ſeries calculorum, quæ adiecta calculo aut quadrato producit maius quadratum.

Vt in



CAPVT XI. 149

Vt uni calculo adjice tres, & fiunt quatuor, quæ quadratum ſunt hoc ſitu vel hoc ſitu vel ſimili.

Huic quadrato quatuor calculorum adjice quinque, & fiunt novem, quæ quadratum ſunt hoc ſitu, vel hoc vel hoc

vel alio ſimili ſitu. Sic quadrato novem calculorum adjiciendo tertium gnomonem ſeptem calculorum fit quadratum ſedecim calculorum. Et adjiciendo huic quartum gnomonem novem calculorum, fiunt 25. Et huic quintum gnomonem vndecim calculorum, & fiunt 36. Et ita ſemper deinceps creſcit minus quadratum in majus, gnomonum adjectione.

Gnomon quàm maximus qui ex calculis marginalibus relictis ſubſtrahi, & in locum vacuum incidere poſſit, dicitur congruus gnomon.

Unde ſequitur, quòd congruus gnomon incidit ſemper in primo, aut ſecundo loco vacuo, qui calculo marginali maximo proximè ſubſtituitur.

His prælibatis extractio quadrata ſic per-

G 3 fici-

Chapter XI: Calculation of the square root.

This process requires one to add counters to the abacus (board) to make square figures. The top of page 149 shows diagrams that explain this process. Begin by placing a single counter on the board (it will actually go on one of the dotted squares). Adding three other counters adjacent (or with blank rows and columns between them and the first one placed) will result in another square figure on the abacus. Similarly adding another five counters to this (with or without the blank rows and columns shown) will result in an even bigger square.

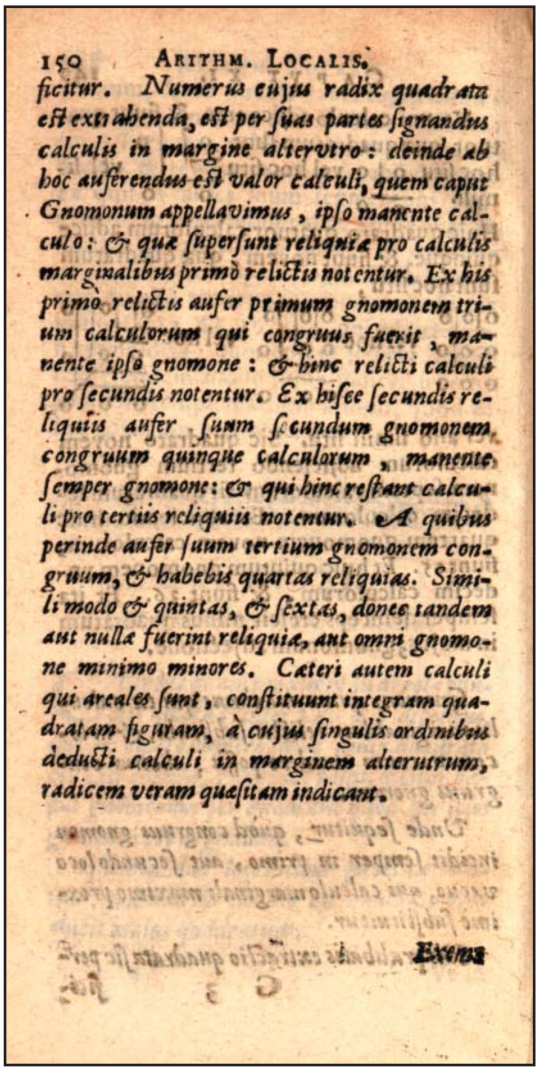
Take the number to be considered and put counters along one margin that represent its value.

From the position of the largest counter in that value, follow the diagonal lines (bishop's moves) across the board until you come to a square with a dot. Place a counter on that square.

Subtract the value represented by this single counter from the original number in the margin.

Add three (five, seven, ... for subsequent steps) to create a square on the board and subtract the value of the added counters from the number in the margin until the number is either too large to be subtracted or there is no space left on the board. You should be left with a large square of counters (perhaps with blank rows and columns between them) on the board.

Move one of the counters in each row of the square to the margin and the positions of these marginal counters will yield the square root of the number.



150 ARITHM. LOCALIS.

ficitur. Numerus cujus radix quadrata est extrahenda, est per suas partes signandus calculis in margine altervtro: deinde ab hoc auferendus est valor calculi, quem caput Gnomonum appellavimus, ipso manente calculo: & quæ supersunt reliquiæ pro calculis marginalibus primò relictis notentur. Ex his primò relictis aufer primum gnomonem trium calculorum qui congruus fuerit, manente ipso gnomone: & hinc relicti calculi pro secundis notentur. Ex hisce secundis reliquiis aufer suum secundum gnomonem congruum quinque calculorum, manente semper gnomone: & qui hinc restant calculi pro tertiis reliquiis notentur. A quibus perinde aufer suum tertium gnomonem congruum, & habebis quartas reliquias. Simili modo & quintas, & sextas, donec tandem aut nullæ fuerint reliquiæ, aut omni gnomone minimo minores. Cæteri autem calculi qui areales sunt, constituunt integram quadratam figuram, à cujus singulis ordinibus deducti calculi in marginem alterutrum, radicem veram quæsitam indicant.

Exemp

CAPVT VNDECIMVM. 151

Exemplum.

Sit extrahenda radix quadrata ex 1238. Numerum hunc signabis calculis in margine altero, utpote dextro, iuxta numeros 1024, 128, 64, 16, 4, 2, ut in inferiori Schemate. Deinde deponatur calculus in areola punctis notata quæ valet 1024, & caput Gnomonum est; quo manente immoto, aufer ipsius valorem ex dictis calculis marginalibus, & supersunt

G 4

Napier provides an example of determining the square root of 1238.

The largest counter is in the 1024 position so the first counter is placed on the dot found by moving down the 1024 diagonal (at the 32,32 position). Subtracting this value (1024) from the original number leaves counters at 128, 64, 16, 4 and 2 (= 214).

Placing three counters on the board to form a square with the first counter but whose value can still be subtracted from 214, results in counters at positions 32,2; 2,2; and 2,32 (whose values are 64, 4 and 64, which when subtracted from the remainder of 214 = 82.

The next square that can be constructed from five counters, yet the values of those five counters still being capable of being subtracted from 82 results in counters in positions 32,1; 2,1; 1,1; 1.2; and 1,32. The values of these five counters total 69 which when subtracted from 82 leave 13 as a remainder.

As there is no more room on the board we have to stop.

Move one counter from each row to the margin (rows 32, 2 and 1) and this value (35) is the square root required, or at least the integer part of it (the actual value is 35.1852....).

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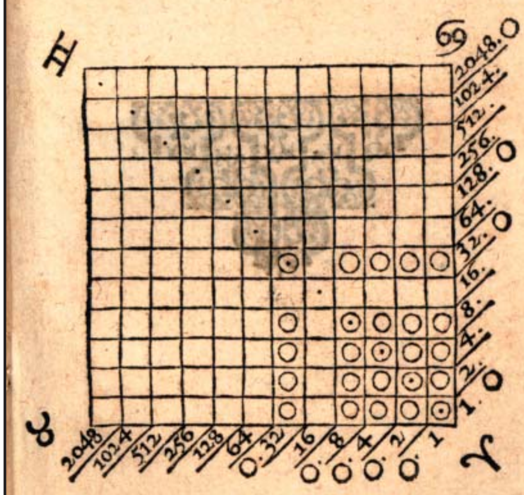
ſunt calculi apud 128, 64, 16, 4, 2, pro reliquiis primis. Ex his primis aufer valorem primi gnomonis congrui trium calculorum in area (ut vides) depoſitorum; & qui ſuperſunt calculi pro ſecundis reliquiis notentur, quæ incident juxta numeros 64, 16, 2. Ex hiſce ſecundis aufer ſuum ſecundum gnomonem congruum quinque calculorum, (manente tamen gnomone in area) & ſupererunt calculi juxta numeros 8, 4, 1, qui additi faciunt 13 pro tertiis & ultimis reliquiis. A ſingulis autem trium huius quadrati ordinum, dirigantur calculi in marginem alterum inferiorem, & hi iuxta numeros 32, 2, & 1 incident, qui additi ſunt 35, radix quadrata quam quæſivimus.

Aliud Exemplum.

Sit extrahenda radix quadrata ex 2209. Numerus is conſtituatur in margine alterutro, v. g. dextro, calculis iuxta 2048, 128, 32, & 1 poſitis. Ab his aufer valorem calculi qui caput gnomonis eſt, quod in areolam punctis notatam 1024 incidit, & ſuperſunt primę reliquiæ iuxta numeros 1024, 128, 32, & 1. Hinc aufer valorem primi gnomonis congrui in area depoſiti, & ſuperſunt ſecundæ reliquię iuxta numeros marginales 512, 64, 32, 1. Ex his aufer valorem ſecundi gnomonis congrui in area depoſiti, & ſuperſunt tertiæ reliquiæ iuxta nu-

numeros marginales 256, 16, 1. Ex his tertiis aufer valorem ſui tertii gnomonis congrui, & provenient quartæ reliquiæ in margine iuxta numeros 64, 16, 8, 4, 1. Denique ex his quartis reliquiis aufer valorem quarti gnomonis congrui, & nihil remanebit pro noviſſimis reliquiis. Radix autem quæſita colligitur ex calculis quinq; lateralibus, quos ſinguli huius quadrati ordines in margine dirigunt: hi enim ſunt iuxta numeros 32, 8, 4, 2, 1: qui additi con-



Napier provides a second example for calculating the square root of 2209 (= 47).

constituunt 47 radicem quæsitam, ut videre est in schemate, quoad pictura patitur; motus etenim calculorum multò faciliùs & certiùs in abaco maiore, & calculis suis mobilibus, quàm in his prælo fixis & immobilibus intelligitur; ut superiùs etiam admonuimus. Atque hic finem ARITHMETICÆ LOCALI imponimus. DEO soli laus omnis & honor tribuatur.

FINIS.

The blank sheet is the recto of the free endpaper.