10 Teaching for understanding

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'Understanding' has several different meanings: knowing how to perform and use mathematics (instrumental and procedural); knowing about usefulness in context (contextual); relating mathematical concepts (relational); knowing about underlying structures (transformable, generalised and abstract); having overcome inherent obstacles. [1]

Levels of understanding arising from a tables exercise

To illustrate these meanings I shall describe an exercise given to a Year 9 class of very low achievers to help them practise the seven times table. It may come as a shock that pupils of 13 and 14 needed such practice, but I shall focus on the structure and aim of the exercise.

Pupils were given a sheet on which they could fill in answers to four columns of calculations. Here is an excerpt, which could be used as a teaching tool in a variety of ways:

. . Pupils filled the first two columns vertically, usually by adding seven each time. Few knew answers without this adding process. No pupil connected the columns and worked horizontally.

In what sense was this exercise enabling them to practise? If the aims of practice are fluency and recall, how does repeated addition help? It is likely that the pupils were focusing on adding seven with the aim of getting the worksheets completed, rather than thinking about the number facts they had generated. Appropriate teacher intervention, restructuring the task to focus on aspects other than filling in the answers, can make a crucial difference to learning outcomes.

Here are some ways in which this task can be used to encourage a range of understandings.

To encourage instrumental and procedural understanding

To recall and use multiplication facts one has to be able to know them individually, rather than only have inefficient routes to find them. This worksheet is physically structured to encourage recursion. To focus on individual facts a different physical layout would be essential, one in which '6 × 7' cannot easily be related to '5 × 7' except in the pupil's head. Some pupils might have a visual memory which enables them to imagine the worksheet layout to get the right answer; some might rapidly reconstruct the answer from a small bank of memorised facts; others may use rhythms of words to aid recall (as in 'seven elevens are seventy-seven'); others might reorganise facts into groups which make sense to them in some other way. Pupils have personal ways of remembering and the teacher has to intervene to get their attention off the page and into their imaginations. The task, therefore, is not finished when the gaps are filled, but after the completed tables have been used to aid memory and recall. To do this, there needs to be focused questioning and further tasks, such as a game in which quick answers are required, repeated over several lessons so that pupils can be aware of their progress.

Can the task be developed to help pupils develop a deeper understanding of multiplicationprocedures? A teacher might draw pupils' attention to the commutativity of multiplication, which always suggests an alternative procedure, and ask when it might be useful.

To encourage contextual understanding

This exercise is purely about number relationships which could be applied in contexts.In primary schools pupils are often asked to tell a story that illustrates a number fact. If teachers choose contexts they need to ensure relevance for the pupils; it might be better to get pupils to provide their own contexts. It is highly likely that some contextualknowledge of number facts might come first with answers being treated as abstract number facts. For example, a pupil might know that 3×7 is 21 from playing darts, or that 7×5 is 35 from handling money. These may seem trivial examples but it is always worth asking pupils what they already know.

To encourage relational understanding

The exercise provides opportunity to relate each number fact to three other expressionsof the same relationship. The pupils' concentration on vertical relationships may lead them to ignore both commutativity and the relationship between multiplicationand division. Intervention by the teacher could redirect the focus towards the horizontal patterns (Watson, 2000) by separating one line from the others and asking 'What do you see?' or 'Where do the same numbers appear?', then offering '7×23' as the start of a line and expecting pupils to complete it. Pupils are almost forced to look at the relationship as a structure. Looking for non-obvious patterns makes structures more important than answers and gives pattern-spotting a powerful role in learning mathematics.

The omission of an entry in the '7×7' line can be discussed to bring out the relationshipbetween this exercise and square numbers. The positioning of the numbers in the last two columns can be discussed and pupils asked to explain the relationship between divisor and quotient.

To encourage transformable , generalised and abstract understanding

Although this is such a simple piece of mathematics, it is worth noticing that it can be generalised. The four forms for each multiplication fact can be expressed algebraically, and pupils asked to transform between them. For example, pupils can pose questions of the form '112×34=3808, so what is 3808 divided by 34?' or 'If $pq=m$, what does q equal?'

At a higher level of abstraction, pupils could be asked to find another operation which is commutative and set up a row of its different representations, and one which is not. This is a further shift, beyond generalisation, towards the abstract notion of binary relationships.

The reader may like to imagine further whether the task could be used to overcomeinherent obstacles in multiplication. The appropriateness of the task for 13-and 14-year-olds clearly depends on how the teacher interacts with pupils about possibleways to reflect on their work.

Task choice, lesson structure and interaction

Each of the approaches described above requires more from the teacher than merely selecting the exercise and helping pupils finish it correctly. In fact, given unmediated without teacher intervention, and without some reflective discussion afterwards, the task does not promote even the simplest levels of memory or understanding. But this is not to suggest that pupils gain nothing from it. Understanding is achieved by a process of personal 'sense-making' by the learner, and a pupil who is mentally engaged with even a mundane task may develop some level of understanding. Conversely, it is possible for some of the above ideas to be treated on a superficial level by the learner, and little growth of understanding to be the outcome. The teacher cannot guarantee understanding, but *can* plan for pupils to have experiences that are likely to lead to higher levels of understanding (or to memorising and fluency, if those are the aims) by structuring tasks and discussion to focus sharply on the desired features. To understand requires mental effort (Newton, 2000) and the teacher has to channel this towards goals of understanding.

Consideration of the simple repetitive exercise above shows that teachers need to be clear about the kind of understanding they wish pupils to achieve, and use strategiesthat are likely to create appropriate learning experiences. This cannot be done by simply choosing 'good' tasks. The task above was fairly mundane but could be used to create purposeless or purposeful activity according to the way it was used in the classroom.

The National Strategy recommends a three-part lesson consisting of a mental and oral starter, a main part and a plenary at the end. It is possible to interpret a fairly traditionalUK mathematics lesson as fitting with this pattern. Exposition could be described as an oral starter because it contains some questions and answers, but the questions would often be procedural and the answers perfunctory; the main part would be working through a textbook exercise; in the plenary answers can be read out and further explanation might be given. Superficially, a three-part lesson structurehas been followed, but only instrumental or procedural understanding has been programmed into it.

Using the tables task, a better three-part lesson might start by sharing ideas about how to fill in the answers, the main part could consist of filling in the answers and looking for patterns within and between columns and seeing if similar patterns occur with other tables, and the plenary could be a substantial whole-class discussion of the relationship between multiplication and division, or a game highlighting rapid recall of multiplication facts which might be repeated at the start of the next lesson. A particularlesson structure, like a particular task, does not guarantee understanding.

Strategies which promote understanding

The following sections describe strategies which are likely to foster understanding. You will probably notice that, although each part starts with different intentions, there is convergence towards practices that overlap all sections.

Teaching for instrumental and procedural inderstanding

Learning new procedures often entails using previously-learnt procedures fluently in complex situations. The teacher has to help pupils sort out what ought to become fluent and what can be reconstructed from a deeper understanding. Here are two examples: it is a good idea to know fluently what 'cosine' means and not have to look it up when it appears in a mathematical context; it is a good idea to know fluently that 48 is the product of several pairs of numbers so that when it appears as the constant term in a quadratic one has a possible starting point for factorisation.

Learning theoretically, combined with frequent opportunities to use what has been learnt so that it becomes fluent in context, seems to be the way we all learn how to use new mental and technological tools. Unfortunately a fragmented mathematics curriculum makes this hard to organise, but learning by heart with the expectation of later recall, and without regular rehearsal through contextual use, does not foster fluency for most pupils.

When the aim is for pupils to know what to do and how to do it, demonstration and procedure-following is useful. But if they have been shown a worked example, pupils will often slot different numbers into the structure they have been shown, like filling in some blanks of a sentence. At the end of an exercise of similar calculations they may have a set of correct answers but not recall how to repeat the calculation on another occasion. Rather than slotting in numbers, the teacher probably hopes that pupils achieve a sense of the procedure and absorb it through using it several times. What actually happens is like drivers being given step-by-step directions by a passenger:they may get to the destination, but may be unable to repeat the route alone on another occasion. Doing something by depending on step-by-step instructions may lead to successful 'doing' but not to learning.

In order to focus on the procedure as the thing to be remembered, something else has to happen; pupils have to generalise about what they are doing. One way to achieve this is to ask them to give instructions for the procedure in their own words. This can be done orally to each other, in writing, as a letter to a mythical younger pupil, as a 'make your own textbook' exercise, or to make up an example and work it through themselves. In subsequent lessons pupils can demonstrate worked examplesfor each other and share the 'inner speech' they use as they do them. Not all studentscan readily describe what they do in words as they may have other ways to 'see' procedures (this can apply even to gifted mathematicians), so I am not suggesting that this kind of activity is essential for everyone, but it does provide reflective action which can aid recall of the procedure. Hiebert and Wearne (1996) found that pupils who were encouraged to develop their own procedures and explain them to each other improved in their ability to learn later procedures, and to recall, and to develop procedures where none had been taught.

Consider teaching the conventions of order of operations in algebraic representation.To some extent this is a set of rules that have to be followed and pupils can be told, 'This is the agreed way of indicating the order in which operations are to be done'. This can lead to a lesson in which it is explicitly learnt and practised as an essential tool.

Another approach would be to explore ambiguities which are resolved by having a conventional order, such as BODMAS1. Such explorations and resolutions are important for pupils who are going to use calculators and spreadsheets extensively. They can find out that choosing which operation is done first affects the answer, so there is clearly a need for agreement. They can devise their own rules by indicating button sequences on a calculator. In the end it is important not to leave pupils with the view that their own notations are as valuable as the international convention … they need to conform. However, BODMAS is not the only way to establish the conventionalorder of operations.

An established teacher, seeing pupils develop through several years of mathematics, can develop language forms to be used when brackets appear. Reading arithmetical and algebraic expressions out loud is a powerful way of relating symbol to meaning. The expression:

$$2(x+3)$$

is often read as 'two into *x* plus three'. The word 'into' is usually associated with divisionso is rather inappropriate. A literal reading 'two, bracket, *x* plus three, close brackets' says nothing about meaning. More meaningful readings could be devised: 'two lots of (pause) *x*-plus-three' or 'all of *x* plus three … times 2'.

Reading out loud can establish useful articulations of symbols, and can also reveal helpful and unhelpful ways of reading mathematics which pupils have already developed. In this example, the distributive law (a fundamental structural feature of arithmetic and algebra) can be embedded in the way the symbols are read. There may be no need to recall BODMAS.

Teaching for contextual understanding

For many, the reason for teaching mathematics in school is so that pupils become numerate out of school. Being able to choose and apply appropriate methods in situationsoutside school is, in itself, a skill that needs to be learnt and practised. Modern curricula take this into account requiring that uses of mathematics be discussed in school, that pupils have experience of problem-solving and multi-answer situations, and that contextual questions are worked on regularly. What often results is the use of artificial contexts in mathematics lessons, in which mathematics has to be used in ways in which it would not really be used outside the class. For example, pupils might be asked to answer questions about filling petrol tanks and calculating prices, when in reality most people might fill up to a fixed value, or let the machine do the calculations. Pupils who can imagine themselves into the situation might be confusedby its unreality, and those who cannot are impeded by the contextual details. Contexts that are relevant for pupils and use mathematics with integrity are hard to find. Perhaps one of the teacher's tasks is to extend the pupils' awareness of possible ways in which mathematics might be relevant, starting with their obvious current interests, but indicating other possibilities outside their experience so far.[2]

In Holland, the 'realistic mathematics' movement uses tasks that attract the interestof adolescents and allow them to work as problem-solvers, so that different pupils might use different mathematical approaches (van den Heuvel-Panhuizen, 1994). Cooper and Dunne (1999) have shown that pupils of lower social classes can respond less well than others to contextual questions used for assessment, even in contexts with which they are familiar.[3] A further issue is whether context should be introduced after techniques have been taught, so that the role of context is to provide a forum for practising techniques, or whether context should be the forum for introducing the technique. The first approach requires abstract learning about simple situations to be applied in real, hence complex, situations (general to particular);the second implies that learning can take place by simplifying and abstracting aspects of real, hence complex, situations (particular to general).

Boaler (2001) reports on a school in which pupils were routinely given complex mathematical questions to explore, and thus became used to having to choose and develop methods for themselves. Examination questions held few terrors for them, because they were seen as just more problems to solve. For example, many were able to choose between multiplication and division on the merits of the question, rather than by looking for cues or guessing. Pupils reported recognising how their approach to mathematics in the classroom was similar to their approach to mathematical issues that arose elsewhere. The pupils did not expect every mathematical situation to have an obvious algorithm and were experienced in having to try things out and evaluate the outcome.

Older pupils who have to choose between integration methods, when some questionscan be resolved several ways, and some by neither, will be disadvantaged if they have not experienced trying, adapting, using past knowledge, making judgements and hence developing a 'feel' for choice of method. If working with mathematics includes analysing complex situations, deciding what is important, choosing appropriatemethods and adapting them if necessary then application of mathematical techniques in context, and in examinations, will be less problematic.

Teaching for relational understanding withinmathematics

Returning briefly to BODMAS, suppose that we try to avoid a rule-based approach and look for opportunities to make links with other areas of mathematics. If pupils had already accepted the distributivity of multiplication over addition (and their inverses) because it had been developed as a feature of their arithmetical work, bracketswill have arisen as a way to express this. For a new teacher, teaching 'order of operations' and knowing nothing of the pupils' past experience, it would be impossibleto start from scratch, but it would be possible to find out something about what they already knew by starting a class with some discussion, or activity, which is designed to reveal the range of understanding and experience they have already, and then to vary what happens next to take account of this prior knowledge.

In a study of methods of teaching mathematics, Askew *et al*., (1997) found that teachers who were explicit about connections between different aspects of arithmetic were more successful than those who taught topics separately. Mathematics provides links between procedures, symbols, contexts, purposes, and concrete experiences, so there are many connections to be made. Since most theories of learning agree that we learn by relating new experiences to what we already know, teaching approaches which exploit this by making it easier to build *helpful* relationships might be more successful than those which leave pupils to form their own, perhaps idiosyncratic, relationships. For example, a pupil who thought that solving linear equations was like simplifying equivalent fractions because 'do the same to both sides' was like 'do the same to top and bottom' may have formed an unhelpful relationship. Without guidance, pupils can assume chance or superficial features of mathematical examples are important, and miss the essential features (Anthony, 1994).

Nevertheless, teaching for relational understanding takes time. Skemp (1976) gives four possible reasons for rejecting a relational approach to teaching mathematics:(i) relational understanding may take longer to achieve; (ii) a relational approach may be too hard since relationships can be more abstract than the aspects being related; (iii) pupils may need to learn to perform the skill or technique, say in science or technology lessons, before they can work on the deeper meanings of it; (iv) a teacher might work in an environment where instrumental approaches are the norm and attempts to work relationally are unsupported. If anything, these pressures are more likely now than in 1976 because of the pace of coverage required by theNC for mathematics and the demands of the assessment system. However, where instrumentalmathematics might be easier to follow, give immediate rewards and produce right answers, relational learning promotes adaptability, easier recall and an atmosphereof growth of understanding and can help pupils become intrigued by the subject in its own right, not merely as a service tool or a set of test hurdles.

To teach for relational understanding requires awareness of what pupils already know, and how it is known. For instance, knowledge of common misunderstandings is useful because some, if not all, pupils will have similar problems and activities can be planned to address them explicitly. In these cases, the relationship being sought is with what is already known, and restructuring of understanding is the desired learning outcome.

Being told by the teacher that a current topic has links with previous work is not enough to ensure that pupils make useful links for themselves. Newton (2000) suggeststhat pupils can be asked to create concept maps and flow diagrams to show how they connect different topics for themselves, and to establish the expectation that topics will interconnect. Connections might be through language ('square' being the name of a shape and numbers raised to the second power); through imagery (points on a plane being seen as coordinate pairs, or complex numbers, which can also be related to vectors); through underlying commonality (rotations of order four having similar structure to addition in modulo 4); through the growing complexity of mathematics (equivalence of fractions relating to simplifying rational algebraic expressions); or through making distinctions (irrational numbers are those which cannot be expressed as ratios). Pupils can be asked, when meeting a new piece of mathematics, if it reminds them of anything else; what existing knowledge they have to use to help them understand or do the new mathematics; what is totally new to them and what is already familiar. The teacher has to be adaptable enough to incorporatepupils' responses into the lesson.

Teaching for transformational understanding, generalisation and abstraction

Much secondary mathematics depends on recognising when to use a technique and being able to use it appropriately. Recognition of mathematical situations occurs when the pupil sees a familiar structure lurking within a problem or question. For this reason, pupils need experience of different ways to express mathematics, and of being flexible with what they know.

Pattern-spotting can help here, because patterns give the raw material for generalisationwhich, in turn, enables us to describe facts, properties and techniques in general terms. For example, if pupils look at a collection of quadratics and their factorisations, they might begin to spot patterns in the coefficients and factors. Talking about these, conjecturing and testing their ideas, can lead to a description of a method of factorising. In the same way, looking at the ratio of certain sides in similar right-angled triangles might lead to conjectures about such ratios, which can be tested and then articulated as rules for finding sine, cosine and tangent. In each of these the teacher has directed pupils to look for similarities in structure by comparingexamples, reflecting on their work in a way which goes 'across the grain' of the work done (Watson, 2000).

To transform between representations is an important tool in mathematical thinking. Working explicitly with parallel representations establishes this as a normal feature of mathematics. Sometimes this is traditional, such as when demonstrating a geometrical proof with a diagram, writing a Euclidean argument beside it and speakingan everyday language version at the same time. Teachers can exploit the power of this by using different representations in conjunction so that they are more likely to be linked by pupils. This example is used by many teachers: every time a relationship with structure $a = bc$ is given, the two other expressions of it are also given: $b = a/c$ and $c = a/b$. Pupils with whom this is an established practice have little difficulty with, say, transforming trigonometric formulae.

An important feature of mathematics is that structures which first appear as generalisationseventually become examples of more general concepts. Some examples of this process of abstraction can be seen in secondary mathematics: counting numbers become examples of rational numbers, which are examples of points on the number line; isolated coordinate pairs become examples of the continuous set of points on lines, and the lines themselves become manipulable as linear equations, which could be members of a system of equations. It would be tempting to say that pupils should not be asked to work at a higher level of abstraction until they understandthe previous level, yet it is also true that some features of an object become clear when one attempts to use it alongside others to fulfil some higher purpose. In mathematics, it can be illuminating to use a concept in a more complex mathematical context, and trying to make sense of complexity can teach us more about simple ideas. There is no obviously 'right' way to treat this issue. In a healthy classroom there would be frequent flow between simplicity and complexity, generality and specificity (Mason and Watson, 1999).

Teaching to overcome obstacles

For formative and diagnostic purposes pupils can be asked to explain how they did some mathematics, thus showing what sort of reasoning led to incorrect, or even correct, answers. This method requires close one-to-one attention and is hence difficultto manage. However, a teacher can use similar methods with a whole class in order to be better informed about a range of misunderstandings which they might have.

There are some common errors or misunderstandings in mathematics that occur so frequently, however the topic has been taught, that they can be seen as inherent difficulties in mathematics. Some of them occur because mathematical terms may have a looser meaning in everyday use. For instance, the word 'regular' applied to shape in mathematics means 'having equal sides and equal angles' but in everyday use it may mean simply that the shape is ordinary or symmetrical in some way. Knowing this potential confusion, teachers can use it as a focus for a lesson, rather than just giving the technical word and hoping that pupils will use it with precision in mathematics. Unless shapes loosely called regular are specifically discussed during the lesson, it is unlikely that pupils will make the required distinction. A related common error is for pupils to believe that the diagonal of a rectangle is an axis of symmetry. Again, it is easy to see why, and even good pupils of mathematics may get this wrong because they see that two halves of the rectangle are congruent, so it feels as if *something* should be said about the diagonal! If the focus for a lesson on reflective symmetry is the rectangle, rather than something less problematic such as a square, it is more likely that this obstacle can be overcome.

The examples just given relate to precise use of language, but there are other sources of common obstacles. The belief that multiplication is repeated addition is very strong because, for much of the pupils' lives so far, that is exactly what it means. To shift to understanding it as scaling requires an undoing of images of repeated addition. In fact, if pupils still have an adding rather than a scaling metaphor, great confusion can occur in ratio problems (Hart, 1981).

To teach for an understanding in which pupils have overcome such obstacles it is not enough just to teach the new meaning and hope that some will 'pick it up'. Bruner's model (1960) of learning concepts is helpful here. He said that learning takes place through manipulating and acting with the concept, then forming an iconic representation of it, such as a mental image, and finally being able to express it in symbolic form. If the objects chosen by the teacher are such that common obstacleswill be brought to the fore (such as working with the rectangle when considering symmetry, or working with scaling factors less than unity when constructing two dimensional enlargements) the iconic understandings have to be adjusted to fit the new enacted experience, and cognitive obstacles are more likely to be overcome.

Key practices which promote understanding

I have shown that significant thought has to be given to how a task is structured and how the teacher interacts with pupils about it. Several strategies have been given which are designed to focus pupils on procedures, applications, relationships and the underlying structures of the mathematics. These strategies all assume a classroom in which there is exploration and discussion, the teacher giving questions and prompts which guide pupils' thinking in useful directions, while recognising the supremacy of individual 'sense-making'. There would be explicit recognition of pupils' current knowledge, and their difficulties would be worked on as a legitimate part of coming to understand mathematics. Pupils would be asked to write about, create examples of and use the technical language of mathematics. They would be offered mathematical situations to explore, some of which would be chosen to create shifts in their understanding.There would be different representations given, and pupils would sometimesbe asked to shift from one to the other, or transform their knowledge some other way. Pupils would be encouraged to see links between different mathematical topics, helped to see what is worth remembering, and helped to remember.

Notes

1 BODMAS is a common acronym for: brackets, 'of', division, multiplication, addition and subtraction.
2 John Mason (in Chapter 17 of this volume) has named this 'the zone of proximal relevance'.
3 See also Chapter 13 by Barry Cooper in the companion volume *Teaching Mathematics in Secondary Schools: a reader*, 2001.

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11 Progression in mathematics

Pat Perks

Introduction

Teaching is concerned with enhancing pupils' development, in fact according to the National Curriculum, their spiritual, moral, social and cultural development. But what is development? As a teacher I should be able to account to other professionals and to parents for statements such as 'Mary is making good progress'. But what does this mean? Does this mean that Mary was at point x in the curriculum and is now at point y, or that she knew x but now knows x better, or a combination of the two or indeed that she is settling well into the class and is prepared to join in discussion? I suppose the Holy Grail for teachers would be that magic sequence for presenting mathematical ideas that would ensure maximum progression in pupils' learning. Questions remain about whether this is a possibility.

…my view of teaching is that of teachers as inquirers attempting to solve pedagogicalproblems. The pedagogical problems are not of the type…such as the use of audio-visual aids, or how to write on the chalkboard, or how to use one's voice; rather they are concerned with more fundamental questions such as the conceptual structure of the subject under study or the most appropriate approachto teaching for meaningful learning.

(Stones, 1992, p. 14)

Teaching is not just a body of craft knowledge, it is difficult to explain and analyse. Why does the material you use with one class seem to result in learning, but with another just causes confusion? Why does a session appear really exciting and useful to some and yet boring and uninformative to others? In trying to identify how best to help pupils learn, to make progress, the teacher cannot just turn to a formula and apply it. You have to identify the problem, find the parameters, work towards the best solution available at the time, then evaluate the solution in terms of the learning of the pupils and refine the solution. Whilst ideas, materials and methods about lessons can be shared, they cannot always be relied on to provide the expected outcome. A brain surgeon deals with one brain at a time, a teacher deals with 30 brains in a class, along with all those raging hormones if you work in a secondary school.

This chapter is concerned with exploring issues related to progression in mathematics– a concept that lies at the very heart of becoming and being a teacher. There are those in public life that think that answers to progression questions ought to be easy, who think that progression is equivalent to acceleration through the defined curriculum. Not so, this chapter is not about acceleration (Gardiner, 2000) but about working with pupils learning mathematics in a deep and meaningful way with all the consequent complexity.

Defining progression

'Progression' is one of the key words in many of the documents and policies for teachers. Teachers are expected to plan for progression, to account for it and to ensure that it happens. A first task is to define the 'it'. What is progression? Does it only happen in the long term? Or can it be accounted for in the short and medium terms? There are two main ways to describe progression, the first relates to teaching sequences and the second to pupil development. As a teacher you need an understandingof these two types of progression – which are often confused in policy documents:

- progression through the curriculum in order to develop teaching sequences; you need an understanding