

**SITUATED COGNITION
AND THE
LEARNING OF MATHEMATICS**

Edited by

ANNE WATSON



Centre For Mathematics Education Research
University of Oxford Department of Educational Studies

**SITUATED COGNITION
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A ground-breaking collection of research and theoretical writing exploring the relationship between school-based learning of mathematics and learning mathematics in other situations. The chapters of this book expose differences between the use and learning of mathematics in school, home, everyday life and the workplace. Some difficulties in transferring knowledge of mathematics from one situation to another are made explicit through the consideration of differences. The ideas in this collection challenge the assumption that school-taught mathematics might be a suitable preparation for adult use of mathematics. Also examined is the possibility that school pupils' experience of using mathematics elsewhere might be harnessed for more effective school learning.

The contributors come from a variety of backgrounds: teachers, mathematics educators, lecturers and researchers, in UK and elsewhere. They have all, in various ways, attempted to apply theories of situated cognition (most notably that of Jean Lave) to classrooms and other situations.

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Centre For Mathematics Education Research
University of Oxford Department of Educational Studies

The Centre for Mathematics Education Research at the University of Oxford Department of Educational Studies is particularly interested in the theory-practice dialectic of teaching and learning mathematics. This focus arises from, and leads to, close contact with teachers, particularly teacher-researchers. A high regard is held for classroom-based research, and the Centre publishes a series of Occasional Papers reporting small-scale studies.

The work of the centre is led by Barbara Jaworski and Anne Watson.

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Foreword

On May 3rd 1996 Jean Lave, a key figure in the social anthropology of people's learning and use of mathematics, led a one-day seminar at the Centre for Mathematics Education Research at the University of Oxford. The seminar was attended by mathematics educators from UK, Europe and beyond, and provided a focus for those already researching aspects of the situated learning of mathematics as well as a stimulus for others to do so.

The following year, April 1997, a conference took place at which some of the related research was presented and discussed. This book started life as the proceedings of that conference, however it was then decided to develop some of the papers further to make a more substantial publication. All the papers which are now included as chapters have been reviewed, discussed and rethought since the conference. In addition, Jill Adler has contributed a significant chapter by special invitation. The collection includes the work of new and established writers, from theoretical and empirical perspectives, and provides an up-to-date overview relating theories of situated cognition to the teaching and learning of mathematics.

In preparing this book I have depended heavily on the comments of those who reviewed the chapters and others with whom I have had long conversations about the contents. Special thanks are due to:

Leone Burton, Madelina dos Santos, Steve Lerman, Jo Boaler, Dhamma Colwell, Peter Winbourne, Alison Price, Tim Rowland, Brian Hudson, John Monaghan, Barbara Jaworski

and to Clare Atkinson and Angela Triner who helped with the administrative, clerical and technical aspects of production.

In the end, however, the editorial decisions were mine alone.

Anne Watson
University of Oxford, 1998

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Martin Hughes has been Professor of Education at the University of Exeter since 1994. His main research interests are in the development and education of young children, with particular interests in learning maths, learning with computers and the role of parents. His books include Young Children Learning (with Barbara Tizard, 1984), Children and Number (1986), and Teaching and Learning in Changing Times (1996).

Stephen Lerman was a secondary school teacher of mathematics in England and in Israel. He is now Reader in Mathematics Education at South Bank University in London and Head of Educational Research. He is President of the International Group for the Psychology of Mathematics Education and was previously Chair of the British Society for Research into Learning Mathematics. His research interests include: philosophy of mathematics; teachers' beliefs; teachers as researchers; equity issues; learning theories, Vygotsky, and socio-cultural perspectives of mathematics teaching and learning.

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Joao Filipe Matos is Assistant Professor in Mathematics Education at the Faculty of Sciences of the University of Lisbon and member of the International Committee of the Group for the Psychology of Mathematics Education (PME). His field of research is learning and mathematical thinking. This is also the focus of the research projects in progress under his co-ordination.

Madalena Santos teaches mathematics in a public secondary school near Lisbon. She focused the research for her Master's dissertation on school mathematics learning seen as participation in a practice. The main topic of the research now being conducted in Cabo Verde (as part of her PhD) is youngsters' learning as a social practice in socially organised settings out of the school.

Anne Watson is University Lecturer in Educational Studies (Mathematics) at the University of Oxford, and co-director of the Centre for Mathematics Education Research. She has taught mathematics in secondary schools for many years. Her current research interests include social justice in mathematics assessment, and the power of interactive strategies in the classroom, recently co-authoring 'Questions and Prompts for Mathematical Thinking' (1998).

Peter Winbourne works at the Centre for Mathematics Education at South Bank University. He taught mathematics for fifteen years in London comprehensive schools before moving into advisory work and thence higher education. For a long time his main interests were in examining the impact of powerful new technologies on the teaching and learning of mathematics, and what mathematics is actually thought to be. More recently he has become convinced

that who learners are, and how they come to develop a sense of who they are becoming, is a much more interesting field of study.

Julian Williams directs, researches and teaches in the Centre for Mathematics Education at the University of Manchester. His current inquiries include the study of learning, assessment and teaching mathematics in schools and the connections between academic and informal knowledge. A current interest in dialogue springs from its significance both to research methodology and teaching methods.

Introduction

WHY SITUATED COGNITION IS AN ISSUE FOR MATHEMATICS EDUCATION

Anne Watson

University of Oxford

During the late 1980s, publication of a paper by Carraher, Carraher and Schliemann (1985) and Lave's book *Cognition in Practice : mind mathematics and culture in everyday life* (1988) highlighted a growing awareness among mathematics educators that the way people learn and do mathematics in school mathematics classrooms is significantly different from the ways they learn and do mathematics in other areas of their lives.

At around the same time Mary Harris (see Harris, 1991) presented many examples of mathematics in out-of-school contexts. She pointed out, for instance, that packaging had intrinsic mathematical properties, and that textile work displayed implicit mathematical principles. The relationship of such mathematics to school mathematics was hitherto largely unrecognised by users and unacknowledged by teachers. This new recognition led some teachers to introduce work-like activities and contextual problem-solving situations into their mathematics classrooms. The aim was two-fold. Firstly, to show that mathematics is relevant to work, this *relevance* leading, perhaps, to increased motivation in classrooms, and secondly, to help students develop the kind of mathematical thinking skills that would *transfer* out of the classroom to help them in their working lives. These aims reflect the concerns of teachers to motivate students to learn and the concerns of politicians and employers that school leavers should be functionally numerate in the socio-economic aspects of their lives. Relevance and transfer will be key threads running through the chapters in this book.

Nunes et al (1993) and Lave, coming from psychological and anthropological disciplines respectively, challenged current pedagogic practice on a more fundamental level within an emerging social theory of learning which would be hard to apply to traditional schooling structures in general and mathematics teaching in particular. In particular, the assumption that increased 'relevance' in school curricula might improve 'transfer' of knowledge and techniques is held up for criticism. In the first chapter of this book Paul Ernest describes and enlarges on a range of epistemological beliefs, which lead to different views of transfer of knowledge, and discusses the implications for mathematics education.

Lave's perspective comes from her observations of tailors' apprentices, shoppers in supermarkets, Weight-Watchers and other practices in which learning

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mathematics and mathematising take place in complex social situations. No explicit teaching (telling or demonstrating) appears to take place, but people learn by taking part in the action, using the tools and language of the situation, gradually becoming more involved as they move from novice to expert, from the periphery to the centre of the action. Learning in such situations is intentional, but is not a separate, formal activity; it is 'an integral part of generative social practice in the lived-in world' (Lave & Wenger, 1991, p.35). Learning is therefore a 'move towards full participation in the sociocultural practices of a community' (ibid., p.29) and is also ubiquitous in all human activity (Lave, 1993). Since the practices within which learning takes place are themselves changing because of the actions of the participants, the knowledge which is learnt is also in a state of change (ibid., p.17).

Such situations are very different from school classrooms in which learning mathematics is the central intentional purpose, explicit teaching takes place and knowledge is not generally regarded as in a state of change. In addition, many of the participants may be reluctant to be there. This contrasts strongly with activities in which everyone, novice or expert, has a similar interest, such as a socio-economic purpose, from which learning emerges as a by-product.

Lave and Wenger (op cit.) feel that the cultural linking of learning to schools gives a limited view of how learning takes place, and yet 'schools themselves as social institutions and as places of learning constitute very specific contexts' (ibid., p.40). In other words, it is by looking at learning in social contexts in general that we will learn more about learning in schools, and not the other way round. Here is a major attraction in her theories for educators, that when we look at classrooms from her viewpoint we see them as social communities in which all sorts of things are being learnt (how to behave in a way that is valued by the teacher, how to be accepted by one's peers, what writing implements are fashionable. . . .) which are not the focus of the teaching. To describe what goes on in a classroom fully one must consider all the actions, thoughts, feelings and environmental aspects within it (Lave, 1993); elsewhere she lists these as mind, body, action and culture. She gives an example of a classroom in which children correctly solve a mathematical problem using their own *ad hoc* methods and discussion (1990, p.321). The teacher assumes that the right answer means they have used the expected conventional method. The teacher believes that the activity was to use the expected method; the children believe that it was to present the right answer. Their practice was not about learning what was expected, but about being successful and surviving in the classroom.

When we look at learning situations outside school with Lave's viewpoint we see in them communities of practice constituted by 'a set of relations among persons,

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activity and world, over time and in relation with other tangential and overlapping communities-of-practice' (ibid., p.98) in which learning takes place through co-participation in the activity, not in the heads of individuals. Further 'a community-of-practice is an intrinsic condition for the existence of knowledge'. As mathematics educators we could dismiss this latter view by saying that one of the important features of mathematics is its abstraction from context. It is often seen as, *par excellence*, the statement of context-free relationships. A belief which underpins the notion that school teaching prepares people to use mathematics in 'real' situations is that the abstraction of mathematics permits learners to transfer knowledge out of and into various contexts appropriately. In practice this is rarely demonstrated in or out of school¹. Lave would deny the existence of such a thing as abstract knowledge at all.

In school all but a few students cannot apply their formal classroom mathematics in their technology or science lessons. Outside school, formal mathematics may not be the most efficient way to proceed with a task. Lave observes (1984, p.93) that shoppers use arithmetic in order to close gaps in their reasoning, the gaps being revealed by a kind of dialectic, iterative relationship between the shopper and the flow of shopping activity. Elsewhere (1988) she shows how mathematics is part of the flow of the activity. Shoppers do not embark on mathematics of such complexity that they would have to stop shopping, but select and monitor their problem-solving techniques in order to continue shopping. The use of mathematics is prompted by real conflicts in a situation, and mathematical solutions might provoke further conflicts rather than resolution of the problem (1990). Colwell² gives three examples which clearly illustrate how mathematics is used and adapted to enable continuity of activities. Hence Lave's theories provide a realistic view of mathematical knowledge which, while not satisfying a purist, adequately describes the lived experience of most mathematics users, and the practical problems of relevance and transfer. Lave develops a theory of learning which applies to *all* learning, in and out of schools, but seriously questions the assumptions of formal teaching and learning, particularly as a means of 'passing on knowledge' in classrooms.

Magajna gives examples to show workplace-generated demand for degrees of mathematical formality which do not necessarily relate to school. An example from my experience is that some caretakers convert Centigrade to Fahrenheit using the rule 'times by two and add thirty' which is adequate and mentally easy but is unlikely to be taught in school because it is inaccurate, informal and limited in scope. Although this latter rule could, in theory, be taught in school

¹ But see Hughes and Greenhough (this volume) for possible examples of spontaneous 'transfer'.

² For authors mentioned in this introduction without references the work referred to is a chapter in this volume.

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the shopping dialectic certainly could not, and some of methods identified by Magajna would have little meaning or motivation in classrooms.

Many adolescents demand relevance in their mathematics lessons, although their world view is limited by their inexperience so they may be unable to judge relevance realistically. At the same time, trying to teach them relevant mathematics ignores the difficulties they will have in transferring and applying school-learnt knowledge to other situations, and divorces the mathematics from the contexts in which it has function and meaning. In those situations mathematics is a situationally-specific tool which contributes to the whole activity; in the classroom it is like learning to use a screwdriver without a screw, or anything into which to screw it.

Lave (1990) herself recognises some of the difficulties in applying an apprenticeship model to school learning; that is, the model of newcomers learning from old-timers while working alongside them.

The gulfin time, setting and activity assumed to separate school learning from the life for which it is 'preparation' is neither reflected nor generated by the process by which apprentices gradually come to be master practitioners. Apprentices learn to think, argue, act, and interact in increasingly knowledgeable ways, with people who do something well, by doing it with them as legitimate, peripheral participants (ibid., p.311)

However, she points to some other aspects of current practice in the teaching of mathematics, for instance investigating and problem-solving (such as those described by Boaler in this volume), as examples of how

it might be possible to learn math by doing what mathematicians do, by engaging in the structure-finding activities and mathematical argumentation typical of good mathematical practice (ibid., p.309).

During the last two decades there have been recent moves in many countries to introduce more activities of these kinds into mathematics classrooms. However, doing what 'real' mathematicians are thought to do is not necessarily the same as learning how to use mathematics in adult life, not as a professional mathematician. For instance, Hudson gives an example of workers filling in a stock-keeping chart using methods of narrow and specific function. Since the structure of the stocksheets and the methods are workplace-specific, the methods may be more easily learnt on the job than theoretically several years earlier in school. School learning is more general in form and needs subsequent adaptation and application to use in other contexts.

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Lerman explores the value and shortcomings of Lave's contribution to knowledge of teaching and learning. Indeed, Lave herself suggests that the function of an apprenticeship model for researchers is as a tool to 'think with'(p.311) and not to be taken as some kind of complete description. Several of the writers in this volume have found that the model has great strengths as a thinking tool.

Adler, referring primarily to Lave and Wenger (op cit.) elaborates this theme by showing that, while Lave's model does not apply to classrooms directly, it does apply to learning *about* teaching mathematics (and hence has application to teacher development) and also focuses attention on the artefacts used in school mathematics learning.

Boaler's chapter suggests that the way schoolwork is structured in one of the schools may enable links to be made more easily with other practices. Hughes and Greenhough research this possibility explicitly by showing that the use of similar structuring resources in different social situations appears to enable very young learners to make unprompted links between their home and school activity. They further suggest that the teacher might be seen as an expert in making links between situations, and hence may help the learner make connections across communities-of-practice. Williams and Linchevski use a disco game and other apparatus as structuring resources in school, and examine how successful they are as learning tools with particular mathematical objectives.

A related matter is the need for 'transparency' in the function of artefacts in the community-of-practice. Adler points out that to be effective at allowing participants to become more expert in a situation the artefacts must be invisible enough to allow access to their function and meaning, and visible enough for participants to use them with effect. The authors in this volume use slightly different interpretations of 'artefact'. Santos and Matos give Saxe's definition (1991) of

historical products that can be conceptual (for example, the scientific concepts), symbolic forms (for example, numerical system) or material (for example, tools)

(P.4)

and examine appropriation of a mathematical artefact by learners in a classroom, drawing also on the ideas of Vygotsky and Schoenfeld. They find that use of the artefact results in more than merely learning how to use it. Noss and Hoyles (1996) have recently developed the concept of 'situated abstraction' in mathematics. This describes the experience of abstracting which people working

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mathematically have, while recognising that such an experience takes place within a socio-cultural environment (ibid., pp.109ff).

Situated abstraction describes how learners construct mathematical ideas by breathing life into the web [of mathematics] using the tools at hand, a process which, in turn, shapes the ideas. Tools are not passive... (ibid., p.227)

The role played by the resources, tools and artefacts in each of the above-mentioned chapters shows that their function and importance would benefit from more research.

Lave's more recent work (e.g. 1996), on which Lerman bases his chapter, attempts to get closer to what happens in classrooms by paying more attention to the life-experience of the individual and how that might contribute to situated learning. Here she attempts to account for differences in learning and also to how learning influences individuals. She sees learners as being participants in several overlapping communities-of-practice, and suggests that the boundaries and interfaces of practices could be a useful focus for research. Boaler shows that learners in some schools are more able than others to relate their classroom learning to other practices, including examinations. She describes two contrasting ways of teaching mathematics. One school uses common UK methods and groupings, the other a problem-solving approach. Lave (1990, p.325) says

Given that the development of an understanding about learning and about what is being learnt inevitably accompanies learning in the more conventional sense, it seems probable that learners whose understanding is deeply circumscribed and diminished through processes of explicit and intense 'knowledge transmission' are likely to arrive at an understanding of themselves as 'not understanding' or as 'bad at what they are doing' even when they are not bad at it (such seems the fate of the vast majority of the alumni of school math classes). On the other hand, learners who understand what they are learning in terms that increasingly approach the breadth and depth of understanding of a master practitioner are likely to understand themselves to be active agents in the appropriation of knowledge, and hence may act as active agents on their own behalf. This is not a ruly process and it is sure to have unintended consequences different from the present unintended consequences of teaching, but perhaps less counterproductive ones than when the question of understanding is simply not addressed in classrooms, as is now generally the case. Such an improvised, opportunity- and dilemma-based learning process may even be a prerequisite for widespread, self-sustained learning.

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Boaler's chapter supports this view. Hudson and Colwell both write about adults who, while showing considerable mathematical skill in their present lives perceived themselves to be failures in the subject at school. In Hudson's chapter some explicit messages emerge about what kind of school mathematics could have been better preparation for their working life. Colwell shows how complex decisions involving mathematics can be made in a variety of ad hoc ways, using situationally-specific knowledge; her chapter also shows clearly that an affective dimension to learning cannot be ignored. She uses, as do Santos, Matos and Magajna, some of the insights of Saxe (op cit.) about culture and cognition as an analytical tool. In particular, Saxe's model of the emergence of goals within situations enables more to be said about individual mathematising than Lave alone can do.

As I have indicated above, Lave (1996) reinforces an earlier theme that different situations do not function in isolation from each other; they may be linked by the way that the mathematics is structured within the situation. The fact that some people can transfer some knowledge between situations focuses her on the development of the identity of individuals within practices, as well as how their experience might influence their learning and contribute towards the situation itself. School children are very much concerned with identity-development, and 'learning, wherever it occurs, is an aspect of changing participation in changing practices' (ibid., p.161). She argues that it is counterproductive to describe school learning as different from other kinds of learning, and instead that

School teaching is a special kind of learning practice that must become part of the identity-changing communities of children's practices if it is to have a relationship with their learning (ibid., p.161)

Winbourne and Watson explore this notion in some secondary mathematics lessons. They adapt the concept of 'community-of-practice' to illuminate certain kinds of classroom learning experience. It is inevitable that tentative application of theory to new fields will lead to some shifts and adaptation of meaning and consequently authors' understandings of the notion of 'community-of-practice' vary slightly. For this book it was felt better to include clarification in individual chapters rather than attempt to force an agreed meaning onto all contributors. Similarly, the reader will have to ascertain the epistemological beliefs (Ernest) of the separate authors rather than assuming a purely Lavian view.

The apparent failure of some aspects of Lave's theories to describe school learning adequately could lead to the charge that the authors in this volume are doing little more than attempt to apply a fashionable theory to their own patch of ground. On the contrary, the authors see theories of situated learning as

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illuminating descriptions of learning in general, and are thus disturbed by the differences revealed between school learning of mathematics and the use of mathematics in other places and practices. In probing more deeply into what situationist theories can and cannot say we learn more about *learning* in ways which are not usually available to a teacher in a lesson. Nevertheless attempts to apply the apprenticeship model (Lave and Wenger, op cit.) to mathematics classrooms, as they are, have necessarily led either to significant adaptations of the theory or tight limits on the field of application.

Unresolved difficulties can be researched further, and an adapted theory might emerge, or, as Adler says:

Perhaps the problem lies in our endless searching for a monolithic explanation of learning. . . . Perhaps learning is, after all, not a unitary phenomenon, and thus not amenable to one all-embracing theory.

This collection represents an overview of some current thinking and research prompted and influenced by Lave. It does not purport to be a complete representation, nor a full critique, of the application of her work to mathematics education. Indeed there are strong indications in this book that Lave's work merely points mathematics educators in new directions, and helps them look at the field in different ways, rather than provides a useful theory in itself.

The chapters

The book opens with an introductory section in which Paul Ernest gives some background to current theories of situativity in learning, as they apply to mathematics and the classroom, in terms of a distinction between explicit and tacit knowledge. Steve Lerman assesses the contribution Jean Lave has made to current thinking in mathematics education.

The second section brings together some work relating out-of-school mathematics to school experience. Dhamma Colwell reminds us of the affective dimensions of mathematical knowledge; Zlatan Magajna outlines some of the complexities of workplace mathematics and makes suggestions for school mathematics; Brian Hudson's study relating to school students in work placements gives more insight into relevance and transfer.

The third section, on school mathematics, opens with Jo Boaler's comparison of two schools with varying mathematical teaching practices; Peter Winbourne and Anne Watson describe types of lesson which might recognise the individual trajectories of students; Madelina dos Santos and Joao Matos focus more closely

on the use of artefacts as structuring resources and ask what makes students argue about meaning. Martin Hughes and Pamela Greenhough explore the importance of structuring resources in two different social situations with very young children, and Liora Linchevski and Julian Williams close this section by relating situated intuitions to differences in students' response when working with various resources, arguing that the master-apprentice model needs to be 'distorted beyond the bounds of utility' to analyse their experimental results, and pleading for incorporation of psychological perspectives in learning theory.

Finally, Jill Adler closes the book by relating Lave's work first to teacher development and then to the role of language as a visible and invisible artefact in teaching and learning mathematics. Thus she makes a link between situated learning theory and discourse theory.

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Section One

Opening Chapters

Chapter 1

MATHEMATICAL KNOWLEDGE AND CONTEXT

Paul Ernest

University of Exeter

This chapter explores the question of what light a modern epistemological perspective can throw on the problems of situated cognition and learning in a social context. The analyses and accounts offered are tentative and evolving, and intended as provocations and reflections indicative of evolving and unfinished thoughts. Some concerns about the problem of the transfer of skills and knowledge from one context to another are expressed, but there is no claim to offer a state of the art survey of situated cognition.

Introduction

The past half century has seen important shifts in conceptions of knowledge including the recognition of the explicit-tacit knowledge distinction (Ryle 1949, Polanyi 1958, Wittgenstein 1953). Explicit mathematical knowledge includes propositions with warrants, such as Pythagoras's theorem. Knowledge of proofs, problems and definitions can also be explicit, but most personal knowledge in mathematics is, I want to claim, tacit. Tacit mathematical knowledge includes methods, approaches, symbolic operations, strategies and procedures which are often applicable to new problems, but are used differently in different situations. For example, the column addition algorithm, proof by mathematical induction, and specific problem solving strategies such as holding one variable constant and examining the resultant pattern of values, are all procedures or methods which, I wish to claim, are largely known tacitly. Hence while the applications of these procedures and strategies are explicit, the more general knowledge underpinning them normally is not.

However the notion of application is a problematic part of the relationship between knowledge and context. Thus an important question concerns the extent to which tacit knowledge is applicable to new situations and what applying it to a new situation might mean. How widely are mathematical procedures and strategies applicable, and when are such applications new? More generally, what features are involved which individuate a context and which distinguish two contexts (i.e. one is 'new' relative to the other) or render two contexts or situations equivalent? (i.e. they are regarded as

mathematically the 'same')¹. I cannot answer these questions, but I wish to signal their import.

First of all I want to make the distinction between explicit and tacit knowledge. Traditionally philosophy and epistemology have focused on explicit knowledge and talked about the warrant for that knowledge. I think we need to accommodate that type of knowledge but also make a space for tacit knowledge. There is a strong precedent for this. Partial parallels exist between a number of dichotomous classifications of knowledge. Thus corresponding to explicit knowledge there is propositional knowledge, which is commonly distinguished from practical knowledge, skills, dispositions. There is also Ryle's (1949) 'knowing that' versus 'knowing how'; there is Polanyi's (1958) and Kuhn's (1970) explicit knowledge versus 'tacit' or personal knowledge; there is Wittgenstein's (1953) explicitly stated knowledge versus the knowledge implicit in 'language games' and 'forms of life'. In our own field of mathematics education there is Skemp's (1976) and Mellin-Olsen's (1981) relational understanding versus instrumental understanding; and there is conceptual knowledge versus procedural knowledge (Hiebert *et al*, 1988). In each of these dichotomies the first of the pair of terms corresponds to explicit knowledge while the second term corresponds to tacit knowledge or 'know how'.

How can this be further elaborated in a way fruitful for the understanding of mathematical knowledge? Philip Kitcher (1984) proposes a model of mathematical knowledge drawing upon Kuhn's (1970) analysis of scientific knowledge. Kitcher calls it a model of mathematical practice, but it does not correspond with mathematical practice in any social sense. So I want to adopt it as model of mathematical knowledge, even though this goes beyond his intentions, which I have done more extensively in Ernest (1997). Table 1 shows the model, and includes Kitcher's components as the first five in the list. I have added two further components at the end of the list because these seem to be important items that are missing from the list, or are only present implicitly. Kitcher does not claim his list to be complete, so it is legitimate to add additional ones if, as I believe, they are needed.

First of all Kitcher includes accepted propositions and statements as mathematical knowledge, and those are mainly explicit. Secondly Kitcher includes accepted reasoning and proofs. Typically proofs are rigorous warrants in mathematics and are fully explicit. Including less formal reasonings opens up the range of items referred to. Accepted reasonings as discursive entities are mainly, if not totally, explicit. Problems and questions are circulated in discussion and between mathematicians and once again, these are mainly explicit.

¹ See Hughes and Greenhough (this volume).

Table 1: A Model of Mathematical Knowledge (Based on Kitcher 1984)

Mathematics Knowledge Component	Explicit or Tacit
Accepted propositions and statements	Mainly Explicit
Accepted reasonings and proofs	Mainly Explicit
Problems and questions	Mainly Explicit
Language and symbolism	Mainly Tacit
Meta-mathematical views: proof & definition standards, scope and structure of mathematics	Mainly Tacit
Methods, procedures, techniques, strategies	Mainly Tacit
Aesthetics and values	Mainly Tacit

Kitcher includes two further areas. One is meta- mathematical views, including views of proof and definition and views of the scope and structure of mathematics as a whole. This type of overview and general views are mainly tacit elements of mathematical knowledge. They are tacit in the sense that mathematicians get a sense of them and build them up incidentally through experience and are not and probably cannot be fully taught explicitly. These elements are usually acquired from experience and are tacit. Kitcher also includes language and symbolism as a further component and these are also largely tacit. Some aspects of knowledge of the language and symbolism of mathematics are known explicitly, but much of their use is tacit, and there are irreducibly tacit elements to this knowledge.

In addition Table 1 includes two further categories not proposed by Kitcher but which are important in discussions of mathematics education. First of all, the methods, procedures, techniques and strategies are important in the context of school mathematics and also in applications of mathematics, but seem to be omitted by Kitcher. Many mathematical methods do not fit under the other categories, and these are mainly tacit elements of knowledge. Maybe some elements of this category are explicit, but like an iceberg, supporting the explicit part is a large body of further knowledge that is tacit. Finally, the second additional category is that of aesthetics and values. In part this is similar to the metamathematical views, but it seems worth singling out as another element, since the values aspects of metamathematical views are not mentioned by Kitcher. Although explicit statements about the aesthetics and beauty of mathematics have been made by mathematicians such as Hardy, most person's positions and feelings about this are tacit, tied into personal beliefs and views which are at best only partly articulated.

The model of mathematical knowledge shown in Table 1 is evidently a broadening and an extension of the traditional view of knowledge as primarily explicit. The wider nature of the elements it includes means that it

is more able to describe the practices of mathematicians and the processes of learning mathematics, since these include tacit elements. It proposes that mathematical knowledge includes a tacit and concrete dimension, made up of knowledge of instances and exemplars; of problems, situations, calculations, arguments, proofs, applications, and so on. This part of knowledge in mathematics, and I also believe in school mathematics, comes from the experience of working with mathematics, and a lot of it is built up tacitly as 'know how' rather than as explicit knowledge. It has already been pointed out by Schoenfeld (1985) and others that mathematical problem solving depends on concrete knowledge of instances, and past problem solutions. Thus there is a mathematical craft knowledge based on concrete particulars and instances which is vital in mathematics and learning mathematics, and much of this is known tacitly, or as knowledge of cases, examples, etc. This new emphasis on the tacit and particular is contrary to the widely held perspective that emphasises the import of abstract and general knowledge at the expense of tacit, concrete and specific knowledge.

It is worth remarking on the parallel that can be made here with the contrast between the scientific and interpretative research paradigms in educational research methodology. One of the features of the interpretative research paradigm is that it valorises concrete particulars and personal knowledge. Thus there is a parallel between the revaluing of tacit and particular knowledge in mathematics, proposed above, and the growing acceptance of the interpretative paradigm in educational research. This is not to denigrate scientific paradigm research nor the value of the generality to which it aspires. It is rather to note the growing value attached to tacit and personal knowledge, and to case studies and particulars in research. This growing new emphasis complements the explicit and general knowledge associated with the scientific research paradigm.

The Social Context of Knowledge and Transfer

Knowledge acquisition skills are socially acquired and knowledge is usually learned in social contexts. I do not think this assertion is controversial, provided that the concept of social context is interpreted widely enough (Ernest 1997). I know a mathematician who studied and learnt university mathematics by reading Bourbaki's Elements in French on his own as a precocious teenager. But I would argue even this solitary learning activity was socially based, because he mastered the language and underlying knowledge socially, that is, in conversation with others, and then exercised them on his own.

Raising the issue of the social context of knowledge involves a way of viewing knowledge that is alien to the received view in epistemology. For knowledge understood as warranted true belief has been viewed as independent of its social context, or even of the context of its acquisition. The context in which someone comes to know has been viewed as inessential to

the status of the knowledge, which is secured by the logical context of its justification, not the contingent social or personal context of its genesis (Popper 1959). Even scientific knowledge which involves knowledge of the world as empirical generalisations transcends the instances of its testing, unless it fails such tests and thus is not knowledge anyway.

Such views represent one way of conceptualising knowledge, and indeed constituted a dominant way. However, they embody a Cartesian or post-Cartesian dualism, one that separates the realm of mind and knowledge, from that of bodies and the tangible world. Thus although logical positivism and logical empiricism reject a literal Cartesian dualism, they reintroduce what I would call a post-Cartesian dualism by distinguishing as ontologically distinct the realm of the analytic, *a priori*, and logic, i.e. that of reason, from that of the empirical, the *a posteriori*, that is from the mundane world of brute contingent fact. Clearly such views have implications for how the relationship between knowledge and social context is conceptualised and how knowledge application and transfer are viewed. In particular, these dualisms view the link between knowledge and context as at best weak, and thus the transfer and application of knowledge as relatively simple.

As my account implies, there are differing perspectives on knowledge application and use and I want to contrast the above view that knowledge is transferable with the view that knowledge is situated. In contrasting these views I do not wish to oppose them as a fixed dichotomy but instead recognise that many perspectives are possible and see their contrast in its simplest form rather as two poles of a continuum. The view that knowledge is transferable sees knowledge as transportable, that is, it travels easily with the possessor, the knower, and is thus effortlessly transferable to a new context. As I have indicated this is a view that is quite widespread, being based on traditional and perhaps unexamined epistemological and ontological assumptions, and is often found in the pronouncements and edicts of bureaucrats and policy-makers. For example, the current vogue for the identification of personal transferable skills in higher education is sometimes based on a conceptualisation of the issue of the inter-contextual transfer as unproblematic. (Below I offer another more defensible, in my view, interpretation of personal transferable skills in terms of problem solving capabilities.) Such views contrast with the notion that knowledge is situated, and that knowledge remains linked to the context of acquisition, representing the other pole of the continuum described above.

The traditional epistemological and ontological views underpinning the notion that knowledge is easily transferable are usually associated with a further notion that there is a unique self or cognising subject separate from knowledge. Several of the other chapters in this volume refer to subjective knowledge and subject-object relations, and it may be that there is often a presupposition that these relations are to do with separation. Namely, that the knower is quite independent of any knowledge, and is an agent who can

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grasp, have, or transport knowledge as if it is an independent entity. Indeed the metaphor of material possession of knowledge (to grasp, get hold of, or carry) presupposes this separation and the very mutable and impermanent relationship between knower and knowledge as a commodity. In contrast, a situated view of knowledge is often associated with a further, different view, and to put it in a neutral way that is consistent with different conceptualisations of the situatedness of knowledge, that is, its indissoluble link with a social context, that there are multiple facets to the self, and that the knower and the known are related and context dependent. There are a number of different ways of elaborating these issues from different perspectives, as these contrasts indicate. Below are six different perspectives on the transferability of knowledge, the relation between knowledge and social context, and the associated concept of self are distinguished.

First of all, there is the perspective that knowledge is universally applicable, based on the assumption that it is abstract and unrelated to context, and typically has the form of explicit propositions or laws expressing relationships. Consequently general knowledge can be applied in contexts by instantiation, through which the specific variables of a concrete situation are interrelated in a structural application of the knowledge, analogous to the process of substituting a particular set of parameter values into equations and formulas. In this way knowledge, like a scientific theory, is fully applicable in new contexts. Furthermore, from this perspective, the knowing self is entirely disjoint from both the knowledge and the context of application, and therefore, for all intents and purposes, can be factored out. This perspective corresponds to what was described above as the post-Cartesian dualism of logical empiricism. It separates the logical realm of abstracted knowledge from the concrete realms of the mundane and subjective (Popper 1979). This perspective does not acknowledge that there are significant differences between contexts, for contexts can only be used to test and possibly falsify knowledge, and not to generate new knowledge (at least not in their epistemological function). Thus the concept of situated knowledge is incoherent from this perspective.

Although this a self-consistent and defensible position, it does not accommodate the broader view of knowledge summarised in table 1, and hence does not admit as legitimate, let alone address, the problems of transfer discussed here. In particular, since knowledge is universally applicable, it makes no sense to say it is transferable or transportable, because there is no origin or location from which to transport/transfer it, since it is located in logical space - not in physical or social space. This perspective corresponds to views of knowledge and learning of traditional epistemology. Knowledge is knowledge because of its justification, and learning is established by the evidence of assessment and both, once validated, are context independent.

Second, there is the modelling perspective which links abstract academic knowledge with the concrete knowledge of specific application contexts dialogically. According to this view, explicit knowledge is fully portable and can be applied in any situation through modelling. Knowledge of contexts of application can also be imported into the context of learning mathematics as a basis for concept development and problem solving. Thus there is a two way traffic between the academic and applications contexts. According to this view, the self is detached from knowledge although new personal knowledge can be induced from immersion in concrete situations, i.e., the context of application.

Third, there is the view that knowledge exists in both explicit and tacit forms, and knowledge which is explicit and abstract is transferable. According to this view tacit knowledge is embedded in certain task specific capabilities. To make knowledge transferable it must first be disembedded from specific tasks or contexts, and transformed into explicit and abstract form. Once knowledge is expressed this way it becomes transferable and transportable, similar to the first perspective. The disembedded and abstracted knowledge is applied and hence re-embedded in a new task context, hence achieving transfer. Underlying this view is the assumption that self and knowledge are separate, but it is understood that tacit knowledge develops as a consequence of experience with specific sets of problems or situations. This perspective acknowledges that problems of transfer exist, but conceptualises transfer in cognitivist terms. Thus transfer is seen as concerning different sets of tasks which vary according to cognitive demand, mode of representation, and perhaps other variables. Transfer of learning thus concerns applying skills and knowledge learned for one set of tasks to another. This perspective thus corresponds to cognitivist views of knowledge and learning.

Fourth, there is the problem solving perspective of transfer that sees a person's higher order problem solving skills as transferable. This perspective acknowledges that both explicit and tacit knowledge exist but it emphasises the individual ownership of knowledge, especially with regard to the tacit knowledge of problem solving. According to this view, the bulk of a person's tacit knowledge including strategic knowledge cannot be made explicit. Instead this knowledge can only travel with a person and is made relevant to a new context of application by the person's immersion in the new context and the cumulative experience of working there. In applying their knowledge in a new context individuals are having fresh learning experiences as well as relating existing knowledge to the new tasks. Personal knowledge is developed through this experience becoming an additional personal resource and knowledge base, extending the person's capabilities without affecting the nature of selfhood. In anticipation of this development, cultural resources from the context of application can be imported into a learning context to prepare the learner for problem solving in the context of application. This perspective conceptualises the problem of transfer primarily in terms of personal cognitive capacities which can be further developed in different

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domains of knowledge application and problem solving. The difference between different domains of knowledge application is not that they are distinct social contexts, but that they are sites combining the application and acquisition of task specific knowledge and capacities with affective factors, that is, goal orientations. This perspective corresponds to a problem solving view of mathematics as fitting well with constructivist views of learning.

The fifth perspective views knowledge as partly situated within the social context of its generation and use. In consequence, some know-how or personal capacities cannot be divorced from their context of origin, but are elicited there by the combination of cues and the personal demands that the context makes. According to this perspective some elements of knowledge can be recontextualised and further developed, as new situated knowledge is created within a further social context, if a knowledgeable person moves across and works in the new context. According to this view the self and knowledge are interrelated. The self has multiple but connected facets each of which is elicited with its associated knowledge and capabilities in the appropriate social context. Thus the social context acts an enabler, providing an appropriate set of personal roles, positionings, interpersonal relationships, expectations, tools, resources, and characteristic activities and tasks, which enables a person to activate a range of capacities and skilled performances. However this complex nexus is socially situated and acts as a whole, and it is inappropriate to think of elements of it being rationally selected and reassembled in another context. Thus the social context is, to a greater or lesser extent, an indivisible whole.

This perspective adopts a situated cognition view of knowledge, learning and transfer. The problem of transfer is not conceptualised as being simply the application of skills learned in one problem set to another. Instead it is conceptualised in social terms: how can the capacities and knowledge and intellectual resources and tools developed for use in one social context be redeveloped, extended and redeployed in another? This perspective approximates to that of situated cognitionists, post-structuralists, social constructionists, and related theorists.

Sixth, there is the more extreme view that knowledge is completely situated and cannot be divorced from its context at all. According to this perspective individuals must be apprenticed within the context of a social activity to master its situated knowledge, and there is little of significance that can be transferred in or out. Discrete segments of the self are developed in different contexts and their resources and dimensions are enabled only in those contexts. Transfer in anything but a trivial sense is eliminated.

Table 2: Different Perspectives on the Transferability of Knowledge

Perspective	View of Knowledge	View of Transfer	View of Self
1. Knowledge is universally applicable	Knowledge is abstract and unrelated to context	In strict terms, there is no transfer. General knowledge has the specific variables of the concrete situation inserted for full applicability	Self entirely disjoint from knowledge and context
2. Modelling links abstract with concrete	Abstract academic knowledge and concrete knowledge of specific application dialogically linked through modelling	Knowledge fully portable and can be applied in any situation through modelling	Self detached from knowledge although new personal knowledge induced from concrete situation
3. Explicit knowledge is transferable	Knowledge exists in both explicit/ abstract and tacit forms.	Explicit knowledge is transferable. Tacit knowledge must be made explicit and abstract before it becomes transferable	Self and knowledge separate, but tacit knowledge developed in contextual experiences
4. Personal problem solving skills transferable	Persons have both abstract explicit knowledge and tacit knowledge, including strategic problem solving knowledge	Tacit knowledge cannot all be made explicit, but is transported with person and made relevant by experience via immersion in context of application	Personal knowledge developed through experience becomes additional personal resource but does not affect core of self
5. Knowledge partly situated	Some knowledge cannot be divorced from context	Some elements of explicit knowledge can be recontextualised and further developed as new situated knowledge is created	Self has multiple but connected facets each of which is elicited with associated knowledge in its context
6. Knowledge fully situated	Knowledge cannot be divorced from its context	Individuals must be apprenticed in new context to master situated knowledge	Discrete segments of self developed in different contexts

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This sixth perspective in pure form probably has few adherents, although it is invoked as a 'straw person' target for attacks on situated cognition or post-modernist perspectives. Virtually all scholars would acknowledge that there are some core elements of selfhood that are transported to any context in which the person is engaged. Otherwise the person could have no knowledge of contexts other than in which s/he was engaged at the moment, and questions would need to be raised about access to personal linguistic knowledge and other resources in multiple contexts.

The six different theoretical perspectives described above are summarised in Table 2.

It should be noted that across this range of different perspectives the concept of transfer of training, learning or knowledge has distinct meanings. In the first and last it has little meaning because knowledge is not associated with context, in the first case, and knowledge cannot be dissociated from context and transferred, in the last. So I shall disregard these extreme cases, which serve merely as markers at the extremes of the range of possible positions on transfer. The four remaining meanings of transfer are those of the applied mathematicians, cognitionists, problem solvers or constructivists, and situated cognitionists or social theorists. These perspectives interpret transfer as:

1. Transfer of learning is application: applying general knowledge to specific concrete situations via modelling
2. Transfer of learning from one set or type of tasks to another - the transport of disembedded knowledge
3. Transfer of learning from one problem situation to another through transport of personal transferable skills (with a person)
4. Transfer of learning from one social context to another through the development of new capacities and facets of self

Just as the concept of transfer needs to be analysed and as here shown to have multiple meanings, the concepts of context and social context need clarification. In particular, what individuates a context, and the criteria for the equivalence of two different contexts is important, as I said above. For until it is determined when two contexts differ or count as the same, it cannot be stated with precision whether successful accomplishment of parallel tasks in the two counts as transfer of learning or simply represents the exercise of the same or analogous skills. However, in my view, the criteria for equivalence and individuation of contexts cannot be uniquely specified, for these will vary according to the perspective adopted including the interpretation of the concept of transfer.

My discussion thus far has been couched in general terms, and has barely touched on mathematics-specific features. Below I consider some of the

mathematics-specific features that arise in a consideration of contextual and pedagogical dimensions of transfer from an educational perspective.

Applications Perspective

According to the applications perspective, modelling builds a link with the applications context. The claim of this perspective is that the problem of transfer is thus overcome. The 'real world' context of the application and the academic or school mathematics context are in a dialogical relation which builds permanent bridges between them, connecting both. First of all, representations from the context of application provide the basis for generating concepts methods and problems in the academic or school

mathematics context, via abstraction and generalisation. So there is a flow from the context of application to that of schooling. Second, there is a flow in the other direction. Abstract mathematical knowledge, concepts, skills and models in the school context are used in applications and verified in the context of application. Ultimately, when the knowledgeable user of mathematics is immersed in applications, the academic/applications concept becomes irrelevant. For mathematics models can be formulated in either. The important difference becomes that between the abstract level of models and the concrete 'real world' level of empirical problems, solutions and data. The applications perspective on transfer, and inter-contextual relations is illustrated in Figure 1:

Figure 1: The applications perspective on transfer and inter-contextual relations in mathematics

<i>'Real world' context</i>	Generation → abstraction	<i>Academic/school maths context</i>
Concrete and applied maths problems and applications	Application ← verification	abstract mathematical knowledge concepts and skills

So this is one way of conceptualising transfer and inter-contextual relations; one that does not see transfer as problematic. It reduces the need for transfer by quilting together the contexts.

Cognitivist Perspective

According to the cognitivist perspective explicit mathematical knowledge learned in the school maths context is transferable to external uses in the 'real world' contexts of numeracy and mathematics. Explicitly learned school mathematics, including symbol systems and computational algorithms, as disembedded knowledge, is applicable to mathematically-susceptible tasks originating in domestic, popular, work and other external contexts. This facility depends on the ability to identify and then work mathematical tasks located in these external situations. This facility is developed by the importation of characteristic elements of these 'real world' contexts. These are external task representations together with some incidental features associated with them to help future identification. The cognitivist perspective on transfer and inter-contextual relations for mathematics is illustrated in Figure 2 below.

Figure 2: The cognitivist perspective on transfer and inter-contextual relations in mathematics

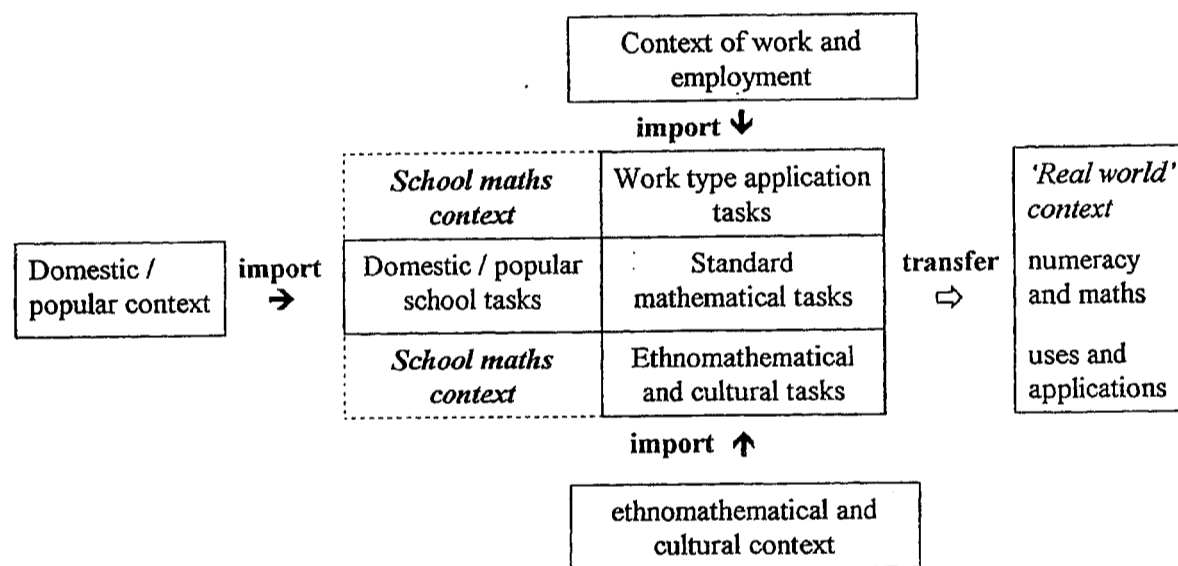


Figure 2 illustrates the transfer of explicit mathematical knowledge from the school context to external 'real world' contexts, which this perspective understands to be unproblematic. This is the implicit model presupposed as underlying progressive mathematics education. The two main contexts in the diagram are the academic school mathematics context and the so-called real world context where numeracy and mathematics are applied. In this model there is the naive assumption that there is such an entity as 'knowledge' - and knowledge related skills are transferred into the real world context. The figure also illustrates three external contexts which are used as sources of task representations imported into the school mathematics context. These are the

world of work and employment, with imported tasks such as calculating tax deductions from wages; the domestic / popular context of hobbies, shopping, etc, with tasks such as modifying cooking recipes, or calculating discounts in sales; and the ethnomathematical and cultural context, with tasks such as drawing Rangoli patterns or Islamic-style tiling patterns. Figure 2 distinguishes the actual contexts which provide the inspiration for such tasks from the subdomain of the school mathematics context containing these task representations. It is widely regarded as good practice within this perspective to introduce a degree of authenticity into tasks by importing detailed representations to provide partial resemblance to the original context-embedded tasks. Thus realistic looking wage slips, advertising brochures illustrating sales goods with prices and discounts, and photographs of

Islamic-style tiling patterns in the Alhambra would serve this purpose. We import representations and elements from the domestic/popular context, and they get transformed, recontextualised or become the inspiration for writing domestic popular tasks for the classroom. Part of the rationale is that this importation provides a conceptual foundation on which mathematical learning is to build, through tapping into meaningful out of school experiences and knowledge. It is also intended that these tasks will be motivational, because out of school activities are purposive and goal directed. The same holds for work context-related tasks, because these are meant to be useful for students in their working lives. Such tasks are also intended to be directly useful, by facilitating transfer into the adult workplace. Thirdly, the importing of elements from the ethnomathematical and cultural contexts of mathematics, particularly informal mathematical reasoning and patterns used in non-European countries, is likewise meant to provide an authenticity to academic school mathematics, and also to facilitate transfer. It is also meant to have socio-political implications, by raising awareness and valorising the products of non-European cultures (Powell and Frankenstein 1997).

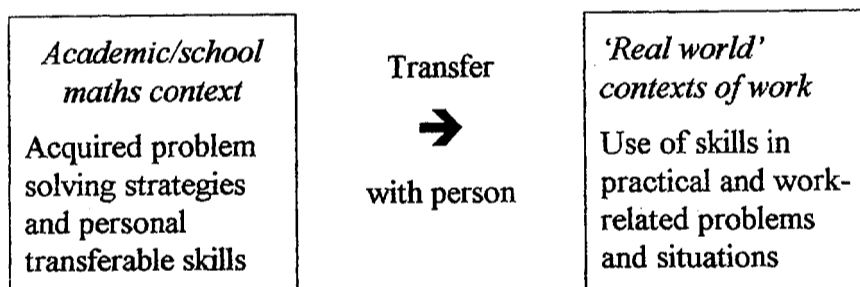
Each of these three types of import involves redundant elements of representation, in terms of the underlying school mathematical task. But this redundancy serves to make the tasks appear 'authentic', that is, as if they were being attempted in the external contexts. As many have remarked, such 'contextualisation' is a form of decoration and can never close the gap between the school activities and authentic context-bound tasks. However it can provide practice in the skills of identifying embedded tasks before applying mathematical symbolisation and procedures. Lastly, Figure 2 shows the central subdomain of the school mathematics context coining standard mathematical tasks. These are 'undecorated' with no direct reference to any extra-mathematical context, and comprise the straight forward application of mathematical procedures to symbolically encoded, that is, routine, mathematical tasks.

Problem Solving Perspective

The problem solving perspective views the most important knowledge for transfer as tacit personal knowledge, namely problem solving strategies and heuristics. This knowledge is acquired primarily from solving non-routine problems in the school context, plus from seeing teachers and others showing solution methods for particular problems. Another possible source of this knowledge might be explicit instruction in mathematical heuristics, although the jury is still out on whether this adds anything worthwhile to knowledge. One of the complexities of problem-solving knowledge is that it can only be learnt from a finite number of exemplars (plus the other possible sources mentioned above) and then somehow it becomes transferable to an unlimited number of examples. Clearly students can only have a finite experience of exemplars from which patterns of heuristics are generalised and induced, providing the basis for transfer. Success in this process seems to depend on the transferable meta-skill of being able to learn from and generalise the knowledge from the instances in the first place. Perhaps this skill corresponds to what Bateson (1972) calls deuterio-learning?

The key feature of the problem solving perspective is that the most significant inter-contextually transferable skills are the personally acquired, personally transportable heuristics and higher-level skills. The problem solving perspective on transfer and relations is illustrated in Figure 3.

Figure 3: The problem solving perspective on transfer and inter-contextual relations in mathematics



An important current issue concerning the transfer of skills which fits under this perspective is that of 'personal transferable skills'. Currently in further and higher education there is an emphasis in this area and curriculum developers are required to specify the personal transferable skills addressed in any teaching module irrespective of discipline, content or aims. For example, at Exeter University six clusters of personal transferable skills have been identified (self-management, learning skills, communication, teamwork, problem-solving, data-handling skills - for further details see appendix) and we are required to list all the personal transferable skills in our courses. It is not clear what is the theoretical basis for identifying these skills. The cognitivist view would be that if we disembed these skills and make them

explicit they can be transferred to and reapplied in new 'real world' contexts. But these skills are primarily strategic, like problem solving heuristics, and such higher level skills by their very nature cannot be made fully explicit. For once strategies are fully explicit and determinate they become algorithms, and lose some of their heuristic quality. Thus it seems more appropriate to consider them with regard to the present perspective, since it concerns the personally acquired and developed strategic skills of problem solving and so on, which it regards as transportable with the acquirer.

Situated Cognition Perspective

The situated cognition perspective is that mathematical knowledge is partly situated and some of it cannot be divorced from its context of origin and deployment. This is what I understand situated cognition, legitimate peripheral participation or the Laveian or social anthropological view to be about (Lave and Wenger 1991). Thus my fourth picture of transfer and inter-contextual relations depicts a number of separate contexts. We have the school mathematics context and some other contexts of which samples are shown in Figure 4, including the domestic and popular context(s) of numeracy and maths use, the industrial and work context of maths applications, the academic university maths context I have distinguished these because they involve different aims, roles, functions and practices, and there is discussion of the problem of transfer from one of these to another. The situated cognition perspective on transfer and inter-contextual relations is illustrated in Figure 4:

Figure 4: The situated cognition perspective on transfer and inter-contextual relations

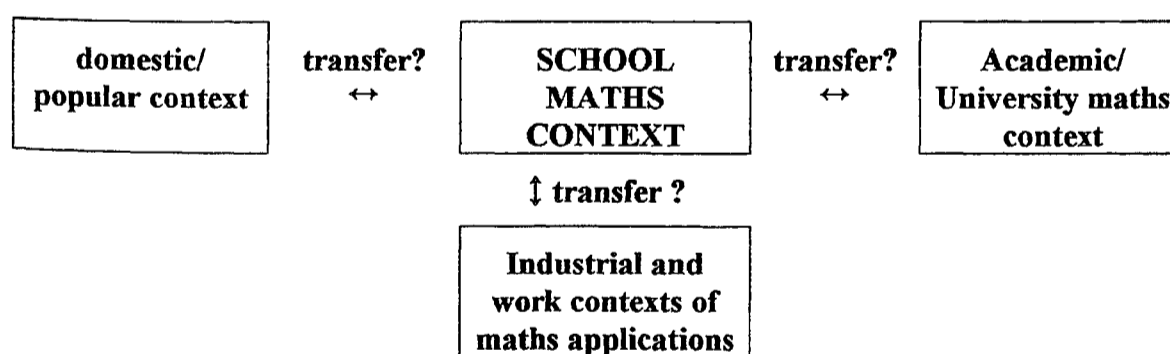


Figure 4 depicts the four contexts shown as discrete social practices, which emphasises the problem of explaining how knowledge and skill are transferable from one context to another. If they are all separate what are the relationships between the discrete social practices? This is a problem for all of the perspectives discussed here, but it is a particularly acute one for a strongly situated view of knowledge. Lave and Wenger (1991) do not discuss much

what a new entrant to a social practice brings with her², but it must include language, personal experience, the ability to learn, and usually the desire to participate in the activities of the social practice. All of these provide the foundation for learning but little in the way of transferable knowledge or skills.

Jeff Evans and others have argued, from a post-structuralist perspective on situated cognition, that a person often "translates" into an unfamiliar social practice: (i) knowledge of signifiers and their meanings within other, more familiar, discourses (Evans, 1999) and (ii) elements of subjectivity including affect (Evans and Tsatsaroni, 1994, 1998). Thus a person in a new social context is in some sense the same person, with some corresponding emotional make-up and signifier resources, but open to the development of new facets of the self through new positionings and relationships.

Conclusion

Knowledge is a multi-dimensional entity as the discussion of the elaborated Kitcher (1984) model shows. Some knowledge takes the traditional form of explicit propositional knowledge, but there is much that is tacit in the knowledge of the learner or the learning community. I have explored a number of different perspectives on the nature of knowledge, its transferability, its relation with the self as conceptualised from the perspectives considered, and its relationship with the social contexts of acquisition and use. These considerations have brought up some of the central problems facing mathematics education concerning mathematical knowledge and context, although my review is only a tentative first attempt to chart the issues and perspectives. Of central importance is the significance that different perspectives attach to tacit knowledge, and their views of how it is related to context.

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² See Lave (1996) for a more recent examination of this issue.

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Appendix

Personal Transferable Skills

We are required to identify our treatment of Personal Transferable Skills in taught courses. Below appear the University's definitions of Personal Transferable Skills.

Self-management:

A Student's general ability to manager his/her own learning development, through the following:

- An ability to clarify personal values
- An ability to set personal objectives
- An ability to manage time and tasks
- An ability to negotiate learning contracts
- An ability to evaluate one's own performance

Learning skills

A student's general ability to learn effectively and be aware of his/her own learning strategies, through the following:

- An ability to learn both independently and co-operatively
- An ability to use appropriate learning technologies, including IT
- An ability to use library skills
- An ability to use a wide range of academic skills (research, analysis, synthesis, etc.)

An ability to identify and evaluate personal learning strategies

Communication

A student's general ability to express ideas and opinions, with confidence and clarity, to a variety of audiences for a variety of purposes, through the following:

- An ability to use appropriate language and form when writing and speaking
- An ability to present ideas to different audiences using appropriate media
- An ability to listen actively and effectively
- An ability to persuade rationally

Teamwork

A student's general ability to work productively in different kinds of team (formal, informal, project-based, committee-based, etc.) through the following:

- An ability to take responsibility and carry out agreed tasks
- An ability to take initiative and lead others
- An ability to negotiate, asserting one's own values and respecting others
- An ability to evaluate team performance

MATHEMATICAL KNOWLEDGE AND CONTEXT

Problem-solving

A student's general ability to identify the main features of a given problem and to develop strategies for its resolution, through the following:

- An ability to analyse
- An ability to think laterally about a problem
- An ability to identify strategic options
- An ability to evaluate the success of different strategies

Data-handling skills

A student's general ability to use data effectively in learning and skills processes, through the following:

- An ability to comprehend data and technique in the context of a student's discipline
- An ability to translate data into words, visuals, concepts, etc.
- An ability to use data as a tool in support of argument

Chapter 2

LEARNING AS SOCIAL PRACTICE: AN APPRECIATIVE CRITIQUE

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Lave's work has been very influential in mathematics education in recent years, offering a view of learning as always situated which has challenged notions of the transfer of knowledge from school mathematics to 'mathematical' practices in the workplace, and of mathematical knowledge as decontextualised. In this chapter her most recent article (at time of writing) is examined and some aspects of her perspective are critiqued whilst also noting and valuing those aspects of her theory which can most fruitfully inform teaching and learning mathematics, and research in mathematics education.

As Jean Lave pointed out, at the Oxford workshop in May 1996, her ideas have undergone significant changes from 1988 to the present. It is therefore appropriate to draw on her most recent work in examining the relevance and significance of her ideas for mathematics education, for teaching and learning, as well as for research, and I will take Lave (1996) as the text which expresses them. In that paper she describes the trajectory of her thinking from distinguishing between formal and informal learning, a theory due to Scribner and Cole (1973) with which she began her early studies of tailors' apprentices in Liberia, to a description and justification of 'what it means to view *learning* as social practice' (p. 150) in her most recent writings. She describes how, in those apprenticeship studies, she came to the realisation that the Scribner and Cole distinction did not work: there is no learning which is not situated and therefore the fundamental assumption that formal learning, which is meant to refer to schooling, is characterised by decontextualised knowledge is not viable. This raises important questions about what is school learning and what is school teaching. Schooling as such has not formed a focus of Lave's research, and whilst previously she has pointed at questions her studies have raised for schooling, she draws directly on examples from schools in her recent work. Clearly her claims for learning as social practice are intended to encourage a questioning of classroom culture and her ideas have been taken up by a number of researchers in mathematics education (e.g. Adler, 1996; Boaler, 1997; Brodie, 1995; Matos, 1995; Winbourne, 1997).

I will suggest that there are some difficulties with her argument. My concerns are threefold: the lack of distinction between schooling as a social practice and other social practices; a down-playing of intentionality in the role of the school teacher's actions as compared with the master tailor; and with 'learning as social practice' as a theory of learning, in Jean Lave's own terms. However, proposing that we see students' learning in formal settings,

as with apprenticeship into all social practices, as 'to shape (and be shaped into) their identities with respect to different practices' (p. 161) is to offer a powerful resource for mathematics education, as indicated by her discussion of 'racialization' (p. 159). Similarly her claim that 'decontextualization practices are socially, especially politically, situated practices' (p. 155) is a key point and contrasts with cognitive theories which place the task of abstraction on the innate potential of the individual, or at least some favoured individuals who develop sufficiently to reach that stage.

Critique

1) Schooling as coercion

There is a clear distinction to be made, in my view, between 'voluntary', possibly life-long situations, such as work practices, societies, cultural groups, and social practices in 'non-voluntary', usually temporary situations such as schools, prisons and hospitals. In the former, individuals participating in those social practices choose to do so, for extrinsic and intrinsic goals and motives: earning a livelihood; gaining entry into a desired group; gaining status and being respected in the community, indeed becoming a person of that occupation, as Jean Lave puts it herself. In the latter non-voluntary situations individuals have no desire to move from the periphery to the centre (Lave & Wenger, 1991), that is, no desire to emulate the practices of the teachers, wardens or doctors/nurses. Even if, in the school situation, a teacher can find ways to engage children so that their own goals and motives can be re-orientated and they participate willingly in the classroom activities, as with the chemistry teacher in Lave's paper (p. 160/161), they are rarely aiming to become chemistry teachers, nor chemists. Indeed the distancing of schooling from other practices, including those of mathematicians, scientists, historians, artists and so on (the subject content labels of the school curriculum), and work practices such as the tailors of Liberia, constitutes a space in which there is contestation for what will take place in schools (Bernstein, 1996). Bernstein refers to 'pedagogic discourse' which is a principle, 'the principle by which other discourses are appropriated and brought into a special relationship with each other, for the purpose of their selective transmission and acquisition' (1996, p. 47). Pedagogic discourse gives rise to a specialised discourse precisely because school mathematics, for example, is not mathematics. Anyone engaged in writing school textbooks is engaged in the pedagogic discourse, not in mathematics. As a principle, pedagogic discourse is the process of moving a discourse from its original site, where it is effective in one sense, to the pedagogic site where it is used for other reasons; this is the principle of recontextualisation. In relation to work practices Bernstein offers the example of carpentry which was transformed into woodwork, and now forms an element of design and technology in schools. School woodwork is not carpentry as it is inevitably separate from all the social elements, needs and goals which are part of the

work practice of carpentry and cannot be part of the school practice of woodwork. Lave's research on the Liberian tailors paints a powerful picture of how a person would become a carpenter. Similarly, school physics is not physics, and school mathematics is not mathematics. Bernstein argues that the recontextualisation or transformation opens a space in which ideology always plays. Thus in the transformation to pedagogy values are always inherent, in the selection, ordering, pacing and so on. Sociologically, then, the teacher never brings 'shopping', or wall-papering a room, into the classroom; it is always transformed. The same must apply in reverse, from the school site to the out-of-school context. Dowling (personal communication) suggests that, from a sociological perspective, transfer can be understood as recruitment (from outside school to the mathematics classroom), colonising (viewing aspects of other practices as mathematical) or as becoming a resource for the mathematics classroom. In each case the process of recontextualisation and the potential for the play of ideology are the critical factors.

The two types of situations, mathematics in and out of classrooms, are similar in that there are senses in which they both can usefully be seen as social practices, but there are significant differences too, which should not be ignored. There is a need, therefore, for a further analysis that takes account of the epistemological differences and the pedagogical implications. I use the term 'epistemological' because of the different nature and function of the knowledge to be acquired by school students as compared with the knowledge to be acquired in a workplace setting. A simplistic description would be that the Goa apprentice wishes to become a tailor, which includes and is built around learning how to do the various things needed to make clothes. The school student's needs in relation to acquiring knowledge, when those needs are not antagonistic to the school's intentions, are somewhere on a spectrum from spending the day peacefully and easily to gaining the right qualifications to pass onto the next stage of becoming an independent adult in the world. Lave's depiction of the school child 'becoming' needs the kinds of analysis of power relations which Bernstein and other sociologists offer (e.g. Dowling, 1995).

2) *Teaching as intention*

The distinction I have drawn above has pedagogical implications too, since the mathematics schoolteacher is not intending that her/his pupils will become teachers or mathematicians. The teacher sees her/his task as some of the following: enabling children to enjoy what s/he is teaching; teaching children what it means to think mathematically, or at least school-mathematically if we take the notion of recontextualisation seriously; helping children gain the best qualifications possible so that they are disadvantaged as little as possible; fulfilling the contract of employment; meeting the school's targets for the league tables; passing on a certain amount of cultural

inheritance, and so on. The school teacher intends to teach, it is her/his job; that is s/he engages in certain activities, which are labelled as 'teaching', which s/he believes will result in students' learning for all or some of the reasons listed above. This is quite different from the situation of the master tailor, whose job is to produce clothes, and if the tailor has an apprentice it is as much to produce those clothes more efficiently and speedily as it is to bring the apprentice in to becoming a tailor. To ignore the intentionality of the schoolteacher is to impoverish the analysis of the classroom. Lave recognises teaching as an intentional activity but refers to it just to argue that it is not a precondition for learning. Her study of apprenticeship indicates that learning takes place in 'the apparent absence of teaching' (p. 151) and she draws on this notion to suggest that school teaching would be better if it were to be constituted around similar relationships. I am suggesting that this ignores some of the particular characteristics of schooling as it is, and is likely to be for the foreseeable future. This is not to say that it is not of great value to attempt to conceive of different forms of schooling. To some degree, one of the schools observed by Boaler (1997) offers an illustration of teaching mathematics built around problem-solving from mathematising outside school situations in which the teacher plays the role of 'master' problem-solver.

There are two aspects of the relationship between teaching and learning that I would wish to develop and both relate to Lave's claim that teaching is not a precondition of learning. At the risk of sounding like the traditional philosopher, I want to say that it depends what one means by teaching. People will not learn things on their own, by which I mean that a child born without human contact, the mythical 'wolf child', will not become a conscious human being. To develop conscious attention, memory, awareness and to gain culture, knowledge etc. are all the results of learning from others. In this sense, teaching is a precondition of learning. (Of course at a later stage one can learn from books, which one apparently does alone, except that the books represent part of human life and the learner-from-books has also learnt how to learn from books. In that students may well be taught to use school mathematics texts in school, this can certainly vary from place to place. An individual can also work things out for themselves, but this may best be understood in dialogic form.) The learning may be in everyday situations or in school or other intentional learning situations. The distinctions between them are important, and the consequences of some of those distinctions are discussed above, but the similarities are just as important. In both cases there is a necessary imbalance, one that is inherently about power relations, but it is mainly in the non-school situation that the learner seeks to become like the 'teacher', mentor, master or leader. Concerning children's social groupings the *telos* of the initiatee is to become like, and to be accepted by, the initiator. In the school situation the imbalance requires further, other analysis (see e.g. Ellsworth, 1989).

Second, I do not wish to suggest that teaching necessarily leads to learning. In some recent analyses of classroom videos of young children in a nursery classroom (Meira & Lerman, forthcoming) we have been describing situations where teaching, in the sense of the teacher's intended actions in the classroom, does not always lead to learning. We have been working with Vygotsky's notion of the zone of proximal development and in such situations we see the zone as not being created in the activity. It is also the case, in my view, that we should admit that some people, including most mathematics teachers, successfully (at least in terms of career outcomes) learned mathematics in what we would now identify and label as the most impoverished way, through drill and practice. Such teachers (I certainly began my teaching career holding that view of good teaching) intend that their teaching will lead to learning by all their pupils. I am not advocating a return to that style of teaching (I suspect it has never gone away in most schools in the UK and probably the world), but that learning theories need to be able to account for that. There is no learning without the creation, or appropriation, of meaning but whether the 'mathematical' meanings are somehow appropriated in those situations, or the meaning is in the successful mimicking of algorithms, is an interesting issue.

3) *Learning theories*

Lave's argument for learning as social practice is developed in criticism of the Scribner and Cole distinction between formal and informal learning and in particular of the characterisation of formal learning as being concerned with decontextualised knowledge. Her earlier writing (Lave, 1988) focused on the paucity of the notion of transfer. This is the view, widely held in mathematics teaching, that formal learning resulted in knowledge which, unlike the firmly contextualised, informal work-place mathematical knowledge, could be applied in a range of situations. She saw the boundaries between each practice as firmly established. Her recent work emphasises the mutually constituting, overlapping nature of the persons we become in the range of social practices in which we act, offering a much weaker view of those boundaries. In her (1996) paper she argues for learning as a social practice to be seen as a full-blown theory of learning.

Lave writes that she finds the following three features of a theory of learning to be 'a liberating analytical tool' (p. 156) for discussing learning as social practice:

1. *Telos*: that is, a direction of movement or change of learning (not the same as goal directed activity),
2. *Subject-world relation*: a general specification of relations between subjects and the social world (not necessarily to be construed as learners and things to-be-learned),

3. *Learning mechanisms: ways by which learning comes about* (p. 156)

She argues that the *telos* of her two case studies, the tailors' apprentices and legal learning in Egypt in the 19th century, is to become masters of tailoring or law and to become respected participants of the everyday life of their communities. It certainly does make sense, as I shall discuss further below, to talk of identities in practice when thinking about how school students 'become', but the overlapping relationships with their teachers and knowledge is only a small part of that. More important to students are aspects of their peer interactions such as gender roles, ethnic stereotypes, body shape and size, abilities valued by peers, and relationship to school life. These are just some of the factors which result in children's intentions in learning very often, if not most of the time, being quite different from those of the teacher (Hallden, 1988). The analogy with Lave's two case studies in terms of *telos* are therefore of limited value, not in relation to becoming a member of (multiple) school communities but in relation to learning school subject content knowledge. The *subject-world relations* are described as mutually constituting each other, a view with which I agree but which requires a clear elaboration of the third of her features, *learning mechanisms*. How do the subject and the social world constitute each other? In fact Lave suggests that the need for learning mechanisms 'disappear(s) into practice. Mainly, people are becoming kinds of persons' (p. 157). Piaget offers a clear elaboration of a learning mechanism, that of equilibration, assimilation and accommodation. Learning becomes the cognitive reorganisation precipitated by disequilibrium. Vygotsky's learning mechanism is mediation, semiotic activity in the zone of proximal development, which is much closer to Lave's mutually constituting subject and object. Vygotsky (1986) proposed that the development of thinking is from the intersubjective to the intrasubjective, and Leont'ev (1981) explained internalisation as the process whereby the internal plane is constituted. I have argued elsewhere (Lerman, 1996) that researchers should be clear about the choices they make concerning the process of learning, both to avoid muddled thinking and also because the theoretical choices we make are realised in the approaches we take to teaching and also to research. A choice between mutual constitution through mediation, or constructivism through equilibration, implies things about the classroom, epistemology and research. Clearly Lave's perspective is closer to Vygotsky's cultural-historical theories than Piaget's individualistic cognitive theory. For Lave's perspective to be a theory of learning requires, in my view, some position in relation to how children (or adults) become, in the social practices in which they act, and there are a number of directions that such an elaboration could take (e.g. Heidegger, Bakhtin, etc.).

Appreciation: shaping identities

Lave draws on Olsen's study of the way that schooling shapes the identities of newcomers to the USA in terms of the 'racialization of social relations and

identities' (p. 159). Thinking in terms of students becoming, in our case, motivated participants in school mathematics, it seems to me, is where Lave's approach is most powerful. The metaphor of students as passive recipients of a body of knowledge is terribly limited, just as is the metaphor of students as all-powerful constructors of their own knowledge, and indeed of their own identities. Lave's focus on the shaping of identity in social practice, extended by an analysis which takes account of the differences between schooling and the practices which she has studied, emphasises the centrality of the social relationships constituted and negotiated during classroom learning. Lave talks of learning as 'an aspect of participation in socially situated practices' (p. 150). Provided we do not expect those practices to be those of the teacher, in our case of mathematics, or the practices of the mathematician, but instead of the practices of the classroom culture, the definition can hold, as is shown by Winbourne (1997), for example. I have mentioned some of the peer issues which are at the forefront, most of the time, for school students, and recognising the significance of these personhoods, or becomings, will help us as teachers to understand (remember?) the nature of the experience of being a school pupil.

From a mathematical point of view, as Lave suggests, abstraction, or the decontextualisation of mathematical actions, has to be seen as another context since there is no such notion as knowledge without a context (Lins, 1994), and one cannot ignore, in particular, the social/ political implications, which I take to mean, for example, the kinds of analysis in relation to social class that Bernstein (1971) offers.

Lave argues here, and elsewhere, that teaching mathematics in schools is usually understood as being concerned with students' acquisition of skills which subsequently can be transferred to other practices and she is, of course, highly critical of that view, both theoretically and in practice. 'Learning transfer is an extraordinarily narrow and barren account of how knowledgeable persons make their way among multiply interrelated settings' (p. 151). Objects, including concepts, have meanings only within relations of signification (Walkerdine, 1988). One of the familiar examples of this is workers not seeing their work practices as mathematical, although a mathematician looking at those practices would wish to say that they can be seen as applications of mathematics (e.g. scaffolders using lengths of pipes which are Pythagorean triples). Transferability, I want to suggest, is a specific mathematical activity, not a decontextualisation of mathematical skills learned, as such, in school. An illustration of this might be the following: Take a class of young children for a walk around the neighbourhood with the instruction 'Observe'. Repeat the walk, this time with the instruction 'Observe and identify as many examples as you can of circles, triangles, squares and rectangles'. The second activity is different from the first because the children are becoming different actors, they are observing with a different

pair of spectacles, those of the mathematician. One might say that they are acquiring transfer-ability. Another aspect of Bernstein's theories which is of great significance to the discussion here is that of what he calls distributive rules. In any context meanings take two forms, the everyday mundane and the transcendental, or immaterial. The former are so embedded in the context that they have no reference outside that context, they are context-bound. In some cultural or social practices meanings are entirely context-bound. Transcendental meanings have an indirect relation to their material base, and this indirect relation allows a potential discursive gap that can become a site for alternative power relations. Without the transcendental, there is no possibility of transfer. Dowling (1995) illustrates how the texts of the School Mathematics Project, which are differentiated according to ability expectations, position low ability readers in the everyday mundane and the high ability readers in the esoteric domain, within the discourse of mathematics. Transfer-ability, from Bernstein's perspective, is the potential to read texts, written, visual, oral or whatever, with mathematical eyes, and this is possible only when one is positioned within the transcendental domain. Dowling's use of the term 'esoteric' rather than transcendental is to emphasise the element of secrecy, the initiation into a society which strongly demarcates those who are not initiates and who are disadvantaged in many ways as a consequence. When Lave writes that 'decontextualization practices are socially, especially politically, situated practices' (p. 155) I understand her to be making the same point as Bernstein and Dowling.

To use Lave's language, '... learning is part of their changing participation in changing practices' (p. 150). Boaler (1997) gives a much more developed analysis of ways in which learners might become successful in viewing a range of problem-solving situations with school-mathematical-eyes.

Learning school mathematics, then, which remains a key to many further life possibilities, can usefully be seen as another context. Since Lave's work, if not before, we have come to realise that it is not decontextualised, which is actually quite clear to the vast majority of people who would say that they have never used school mathematics since leaving school. It is certainly not decontextualised in the sense that mathematics is essentially a set of cognitive structures in the mind, 'built' in school, which can then be applied in a range of problem-solving situations. I remember being offered a job as a mathematician in a team of scientists modelling the pollution of a reservoir, shortly after completing a first and then a second degree in mathematics. I searched my memory for any course I had taken entitled 'Modelling Pollution'; since no such course came to mind I was convinced I could not do the job. I could pass examinations, my knowledge was contextualised to that end, but I had no tools for transfer. Transfer-ability, however, can be learned, when it is seen as a context, as a particular range of mathematical activities

and as a way of seeing the world, and therefore as another way of becoming: becoming a mathematical person.

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Section Two

Mathematics at Work

Chapter 4

FORMAL AND INFORMAL MATHEMATICAL METHODS IN WORK SETTINGS

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In considering the established distinctions between formal learning and informal mathematical activities this chapter focuses on the mathematical methods that are used in formal and informal mathematics. In particular, some properties of informal methods, which are understood as intuitive context-linked methods, are analysed in a work setting. Conditions for using formal and informal methods in work settings are considered and some implications for mathematics education are suggested.

Introduction

One of the most important contributions of theories of situated cognition [e.g. Lave, 1988] to mathematics education is to point out a disconnection between school mathematics and mathematics that is performed in out-of-school settings. In a sense, every practice develops its own mathematics: working practices, school mathematics or mathematics in other school subjects and various everyday situations give rise to specific ways of reasoning mathematically and to specific ways of doing mathematics. Participating in a practice implies an adoption, or at least an interaction, with the mathematics of that practice.

Mathematical practices are sometimes classified as either formal or informal. This supposed distinction is not always clearly stated. Shirley (1995) views *formal* mathematics as pure or practical mathematics which

is taught in schools and universities and continued in mathematical research.

On the other hand he argues that *informal* mathematics includes recreational mathematics and everyday mathematics which

takes place in our everyday lives, in basic counting and arithmetic as well as in intuitive geometry, record keeping, and simple problem-solving algebra. Children usually learn everyday mathematics in elementary school and refine it in middle school.

Thus he implies that formal and informal mathematics can both be taught in school, the former being a development based on the latter.

A slightly different view is that formal mathematics is usually related to the mathematics as it is learnt at school, while informal mathematics comprises

various out-of-school practices and essentially does not transcend our experience. Lindeskov (1991), for example, speaks of 'everyday knowledge of mathematics' as a close synonym of informal mathematics which is

... sensed by the individual as factual knowledge. In this regard the term 'everyday knowledge' is in opposition to the term fantasy. What we sense as factual knowledge we do not question or further investigate, we simply trust it and build on it, sometimes as tacit knowledge without being aware of it.

Similarly, learning itself might be classified into formal and informal learning, but there are various ways of understanding this distinction.

In the following sections I shall analyse whether it is plausible and useful to discriminate between formal and informal mathematical methods, and how such a categorisation may relate to understandings of formal and informal mathematics and to ideas about formal and informal learning. I shall try to find conditions which imply the use of formal *and* informal methods in work settings. Using my fieldwork experience in a glass factory, I shall illustrate the occurrence of formal and informal methods in a work setting.

Formal and informal learning

It is not my intention in this section to define formal and informal learning but rather to examine various possible understandings of these terms. As I shall repeatedly point out, in practice various criteria for distinguishing between formal and informal learning usually coincide.

A natural way of thinking about formal and informal learning of mathematics is to consider the *setting* in which the learning occurs. In this sense formal learning of mathematics might be related to institutionalised education, while learning that occurs anywhere out-of-school is informal. Another important distinction is the *purpose* of learning. In formal learning the aim of the activity is usually related to the object of learning (e.g. one does not work on preparing for an examination and, by the way and unintentionally, learn some mathematics). On the other hand informal learning often occurs spontaneously, without a specific intention to learn, and when the learner's attention is directed to something different from the object of learning, e.g. to completing some task. These two views on formal/informal learning coincide to some extent with each other since people are supposed to do mathematics in schools for the purpose of learning and (usually) to use mathematics out-of-school as a tool, for extrinsic purposes.

For instance, Wertsch (1985) views a difference between learning in school and in the workplace from the point of view of the purpose behind the activities. In work-based activities the main aim is productivity, it is desirable that mathematical activities are done quickly and, above all, without error since errors can be extremely costly. On the other hand the aim of school

learning is understanding mathematics, or at least a skill-related mastery of the topic learnt. The correctness of the methods is stressed and errors are treated pedagogically and sometimes are even intentionally induced. Deliberate learning does not occur only in schools. Wertsch, for example, mentions the interactions of parents with their children when helping them to execute a task: parents sometimes act 'as masters' and directly help the children (who may thus learn by watching and imitating) to accomplish the task, and at other times act 'as teachers' and, by asking appropriate questions or by breaking the task into sub-tasks, try to help their children to learn how to do the task. In this sense, both formal and informal types of learning may occur in work settings too, but most of the learning that occurs during a production process is informal.

Formal and informal learning differ in a whole range of attributes. Harris & Evans (1991) and Masingila (1996), among others, report a comprehensive list of differences between ways of learning. The differences range from types of emotional involvement to social group structure, from the content of learning to the mechanisms of learning and the language used. Some of the differences may be attributed to the different aims of activities, others are probably consequences of the institutionalisation of formal learning. Indeed, the origin of the differences is rather immaterial since the types of learning and their attributes become gradually and inextricably linked to the setting and activity in which they occur.

Formal and informal mathematics

A distinction between formal and informal mathematics has been discussed by several authors. Streefland (1992), for example, defines informal knowledge as

knowledge acquired outside school or knowledge that cannot be considered as the effect of a teaching-learning process aiming at suchlike knowledge.

In other words, informal mathematics is informally-learned mathematics, if we consider the purpose of the learning situation. Such a distinction is deeply subjective and, in practice, it is hardly usable, for it may be impossible to find out where and how a particular mathematical knowledge was learnt. Even if it is possible to locate the learning situation, this may express no more than a tautology. Furthermore, as pointed out by Saxe (1995), the knowledge acquired in different settings may interact. A similar difficulty arises if one adopts a view that informal mathematics is based on making sense of experience (and thus usually learned spontaneously and, possibly, out-of-school) while formal mathematics is obtained by structured appropriation of scientific concepts (usually during formal learning).

Nunes, Schliemann & Carraher (1993) give more subtle observations regarding the distinctions between two possible views of mathematics. They

use a formalist view of mathematics as starting with representations that make possible the use of formalisations; one's subsequent concern is with the relationship between the representations. In this specific sense formal mathematics is 'decontextualised', although it is always linked to the social context in which it is done (e.g. school). On the other hand, informal mathematics is embodied in particular activities and situations, with adopted models which enable the use of mathematical representations. In informal mathematical reasoning people make use of pragmatic reasoning schemes and, using the meaningfulness of the represented situations, they can control the correctness and reasonableness of their postulational (formal) reasoning. Such a distinction between formal and informal mathematics is relatively easy to apply. However, in this sense, learning school mathematics could comprise learning formal *and* informal mathematics, as Shirley (op. cit.) may have been suggesting, but with attention to building bridges between them rather than assuming a cognitive hierarchy.

Formal and informal methods

In this section I turn my attention to mathematical methods. In particular I consider whether it is plausible to speak of formal and informal mathematical methods. Obviously, one can relate formal methods to formal learning or to formal mathematics. Note that in the previous sections I did not specify a distinction between formal and informal learning or formal and informal mathematics, I simply pointed out different understandings of these distinctions. In this sense a distinction between formal and informal methods depends on one's understanding of the differences. Moreover, as I have mentioned, to some extent the various distinctions coincide: an important aim of school mathematics is to develop mathematical ideas that transcend our experience and to think in terms of these ideas (e.g. number, variable, geometric objects) and the methods used are intended to promote understanding rather than being effective in particular practices (Sierpiska; 1995). On the other hand the methods used in various out-of-school mathematical practices tend to be effective and have to take peculiar context-related characteristics into account. In this sense we can think of formal methods as those compliant to the (mathematics) classroom cultures and informal methods as those compliant with various out-of-school cultures. Such a distinction can still be ambiguous, for classroom cultures vary and the same mathematical method can be used by the same person in the school and in out-of-school practices (e.g. calculating the area of a circle using the standard formula).

Each social and economic practice which involves mathematics develops its own mathematical practices which are accepted/adopted by the participants (Lave; 1988). Moreover, as Ernest (this volume) points out, much mathematical knowledge (in every setting) is tacit. Participating in a practice

means accepting, among other things, the mathematical methods of that practice, but the methods may very well be the same in different settings.

A better approach (and one which I use hereafter) is to consider informal mathematical methods as being self-developed, by individuals or communities, through experience. In this sense they are intuitive methods (Lembke & Reys ; 1994). As Fischbein (1987) notes, intuitive methods are characterised by self-evidence and immediacy, and are distinguished by extrapolation and globalisation. Such a characterisation of informal methods does not coincide with the above analysis of the formality of learning or of formal mathematics itself. In school setting methods of both types are common (the interactions between them, including the possible misapplication of extrapolation and erroneous globalisation, present a real matter of concern for the teachers, but I shall not pursue these points further). Also in work settings both formal and informal methods can be observed: a method or a formula that is applied without having an insight why and how it works is thus a formal method, while a spontaneously-generated method is clearly informal. Note that a formal method, using this distinction, may very well be learnt informally on the job (e.g. drawing a right angle using ropes of length 3, 4 and 5 units). Examples of formal and informal methods in out-of-school settings have been described by various authors (e.g. Masingila; 1993, Millroy; 1992, Lave; 1988, Nunes, Schliemann & Carraher; 1993).

The distinction between formal and informal methods is certainly not clear cut, and does not clearly relate to formal and informal mathematics, nor the ways in which it was learnt. The distinction is subjective to some degree and there are methods for which it is difficult to decide whether they are formal or informal. Rather than refining a set of criteria for a distinction between formal and informal methods, therefore, I shall state some common traits of informal methods in work settings.

Being 'self-evident', informal methods have *conceptual simplicity* and are based on experience. The methods are often context-specific on an objective level in that the methods are suitable or they work in certain specific contexts; they are also context-specific on a subjective level in that the practitioner applies and works out the method on objects from a particular context, not on mathematical representations (Nunes, Schliemann & Carraher; op cit.). Furthermore the methods have *algorithmic simplicity* (otherwise they could not be self-evident) and are often *non-deterministic*, for finding a deterministic solution to realistic problems with several more or less specified constraints is usually not easy or reasonable. Informal methods are thus often based on simple and easily grasped strategies like trial and error, local simplifications, 'brute force', and some sort of adaptive strategy. Such methods may appear to be ineffective if viewed algorithmically. However, if they accompany a degree of expertise they may be quite effective. Another important property

of informal methods is *transparency*. Informal methods are commonly 'transparent' to the users: the users often invent them, they may be capable of giving some sort of proof or argumentation for the method and they exert a degree of control over the method. I consider all the above mentioned properties of informal methods follow from the fact that these methods are self-evident and based on experience.

Formal and informal methods observed in a glass factory

I shall now describe some mathematical methods I have recently observed in a workplace. The research took place in Slovenia at a small factory that produces moulds for complex-shaped glass containers. The working methods of six practitioners, all with vocational school training for machine technicians, were studied using a variety of methods for 3 weeks. They all used various computer programs for the design and production of the moulds on numerically controlled machines. During analysis of the study it became possible to describe three types of observed mathematical behaviour which illustrate that a straightforward distinction between formal/informal behaviour is unrealistic:

1. Using a given formal method

The practitioners often had to design a bottle of a given volume and sometimes of a just roughly specified complex shape. One way of designing the bottle was to draw (on a computer aided design system) the imagined (or designed) horizontal sections. They automatically obtained the areas of the horizontal sections, and (using a spreadsheet) calculated the volume of the part of the bottle between each two adjacent horizontal sections using the formula

$$V = \frac{h}{3} (A+B+\sqrt{AB})$$

where h is the distance between the sections, and A and B are the areas of the two sections. The practitioners were not able to say who told them to use this formula - they claimed that it was 'a shop-floor tradition'. They were well aware that it is an approximation, but they did not relate it to the volume of a truncated prism or to Simpson's integration formula. A practitioner explained that the volume of part of the bottle between two horizontal sections could be approximated by

$$\text{vol} = \frac{h(A+B)}{2}$$

which, he claimed, is essentially the volume of the prism. He said that the formula they use is just a better approximation. I consider this method, and

in particular the calculation of the volume between two horizontal sections of a bottle as an example of using a given formal method in a work setting.

2. *Solving unexpected mathematical tasks*

It regularly happened that when the bottle was designed by horizontal sections, its volume, calculated as described above, was not the required one, i.e. not in the admissible margins. In this case the size of the bottle (and at time also its shape) had to be corrected. Sometimes the necessary change of the volume could be obtained easily by dilating the shape in one, two or three dimensions by an appropriate factor. However, in most cases the designers had to observe specific restrictions regarding the shape or certain dimensions of the bottle. Thus, to obtain the required volume, they regularly used a very simple method based on trial and improvement: they repeatedly made aesthetic changes to the horizontal cross-sections (modifying their areas but trying not to change significantly the overall shape of the bottle and to observe possible restrictions) until the calculated volume was correct. Since the method of calculating the volume was approximate and since at this stage there was no reason to obtain the exact volume of the bottle, they found their method reasonable. They solved the mathematical problem of adjusting the volume of the bottle using a rather simple and apparently non-efficient method. However, for the practitioners the method was transparent and they had a complete control over it - and this was essential for during the modification process they could take account of all kinds of restrictions. Thus, to modify the volume of the designed bottle an informal method was used.

3. *Tasks with no reasonable mathematical solutions*

I also found evidence of mathematical tasks where an explicit mathematical solution could hardly be given in advance of the task. Perhaps the most interesting example was making the mould for the designed bottle. To put it simply, once the bottle was designed, the shape of the bottle had to be cut out from the centre of a mould. This was done in several steps and the final volume depended on each of them. Some of the production steps were not completely machine-controlled (e.g. polishing) and each step had its own tolerance. In practice, in order to obtain the desired volume of the bottle there was no point of mathematically elaborating in advance each step. Instead the practitioners calculated and measured the volume of the cut part of the mould several times. Each step depended on the result of the previous step (i.e. measured volume). Using such an adaptive procedure the practitioners were able to obtain the mould for the bottle of the required volume. In this case a mathematical solution could not be given in advance, for the production of the mould involved an approximate calculation of the volume of the original bottle as

well as one or more steps with no complete control on the cutting process. Thus to obtain the correct final volume cut out from the mould the practitioners used an adaptive technique, based very much on their experience.

Formal and informal methods in work settings

These methods of work mathematics, though sometimes idiosyncratic, are part of a practice and clearly differ from the methods that are commonly learnt at schools. As I have mentioned in the previous section, in work mathematics both formal and informal methods are used, and in this respect I claim that at least three cases, illustrated by the above examples, should be considered:

1. *The occurrence of a mathematics related task is expected and a method of solution is provided.*

If the task is known in advance it is reasonable to prepare and arrange a solution or a method of solution for the mathematical task (i.e. provide formulae, tables, graphs, computer programs, etc.). Obviously a practitioner must have a sense of the quantities involved in a calculation, but may not understand the mathematical tasks s/he carries out and often, if everything can be foreseen, no mathematisation is desired since it is time consuming and error-prone. In this sense the practitioner is doing formal and non-transparent mathematics.

2. *The occurrence of a mathematics related task is not expected and the practitioner is supposed to find a solution.*

In this case the practitioner may revert to school-mathematics or (and more probable, according to situated cognition theory) s/he will choose or invent an informal method. The practitioners in general know and understand the object of their activity but, for many of reasons (lack of mathematical knowledge, lack of modelling skills, time pressure and necessity to avoid errors), they prefer some 'transparent' method.

3. *The occurrence of a mathematics related task is expected but for some reason no solution is provided.*

Such occurrences are not as rare as they may seem. Sometimes the task, although it is related to mathematics, can be solved with non-mathematical methods, sometimes the mathematical solutions are so complex that there is no point in using them and sometimes there are no reasonable deterministic algorithms to obtain the solution. Tasks related to the 'travelling salesman problem' or the 'knapsack problem' have no simple algorithmic solutions and informal methods based on experience may give reasonably good solutions. (The first problem is to determine the order in which a salesperson visits a number of cities and travel the minimal

distance. In the second case one is given a knapsack with limited weight capacity and a set of objects with known weights and prices. The problem is to find which items to put into the knapsack so that it will hold the greatest value. Both problems have a number of variations. It has been proved that they have no 'simple and fast' solution (Garey & Johnson; 1979). An example of such an activity in a milk processing plant is described in Scribner (1984). Another example of an operation which is done regularly by tailors but is not easy to treat mathematically is to arrange optimally (or nearly so) templates on a piece of cloth. In solving such problems mathematicians often make use of appropriately formalised informal methods (e.g. Monte Carlo methods, various adaptive methods, genetic methods).

Possible implications for teachers

I have listed three different cases of using mathematics in a work setting. In the first case it can be expected that the practitioner will use a formal method, and in the third case there is little point in using any explicit mathematical procedure. Both situations are predictable and the (appropriate) ways of working them out are part of the practice. Thus, the practice itself implies whether a formal or informal method is to be used. In the second case, in my view, the practitioner can sometimes choose between formal and informal methods. When a mathematically-literate practitioner, for example, has to solve a rather simple and unforeseen problem the decision between formal and informal methods, I believe, is at least sometimes a rational one, based on some criteria, e.g. allowance of error, 'transparency' of methods, uniqueness of occurrence of problem, self-confidence. In my observation in the glass factory I observed several such events: in most observed cases the practitioners were clearly aware of one or more formal methods (sometimes a school-learned method) and of one or more informal methods. The mechanism of choice is not yet clear to me. I have seen them try to use a formal method and they always carefully *check* (by measuring on the computer or the real pieces) whether the results are correct. But I have never seen them *reasoning* about formal methods. On the other hand they reasoned about informal methods and used them, especially when the formal ones, for some reason, did not give the desired result.

The difference between checking results and reasoning beforehand is interesting. Practitioners have a good knowledge of the working processes in which they are involved but, according to my observations, they do not reason about the formal mathematical methods they use in their working practice. For many reasons it is not sensible or possible to relate in schools various formal methods to future working practice. Perhaps it is reasonable to establish such specific (context related) connections after some years of working experience. Such knowledge may help people make better sense of the working activity. An example of such on-the-job learning (and sense-

making) is given in Noss & Hoyles (1996) and I have personally had the opportunity to teach geometry (required to design complex geometric shapes on computers) to experienced CAM technicians who had no difficulty in relating it to and integrating it with their activities.

In school mathematics formal methods are often preferred since informal methods can be learnt in context out of school (Sierpiska, 1995). On the other hand Masingila (1996) and Boaler (1993a, 1993b) argue that it is necessary to work on informal mathematics in school in order to make sense of the school mathematics by connecting it to informal activities. The widespread use and availability of computers, their increased computational power, the interactivity of the programs used, and their ability to simulate makes possible the use of informal methods in cases where the formal methods were previously preferred. The fact that, during the glass factory study, the practitioners in work settings were observed to use formal mathematical methods but to reason only about informal mathematical methods makes me believe that students would benefit from learning about more sophisticated 'informal' strategies, like Monte Carlo methods and other conceptually simple computational strategies. The teaching of such methods could involve prior reasoning, choosing appropriately, checking efficacy, comparing solutions and so on.

Conclusion

Although various mathematical practices might legitimately be considered as separate from each other on the subjective (context-specific) and objective (abstract-formal) levels there is no clear distinction between the mathematical methods that are used in formal education and in various out-of school settings. Informal methods, considered as intuitive methods applied to context-related objects, are used in both school and out-of-school settings, and the same holds for formal methods. I have described some properties of informal methods in work settings and I have indicated some conditions where formal/informal methods can be expected in work settings. School mathematics is usually based on learning formal methods and there is a debate about the role of informal methods in school learning process. Perhaps more emphasis should be put on the criteria for choosing between formal and informal methods in solving problems in school and in out-of-school settings.

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Chapter 5

LEARNING MATHEMATICALLY AS SOCIAL PRACTICE IN A WORKPLACE SETTING

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In this chapter a small-scale case study involving 15-16 years old secondary school students participating in a vocational module under the General National Vocational Qualification (GNVQ) scheme is reported. The development is a pilot study involving experience in the workplace in a small-scale light engineering context. An initial aim of the study was to explore the potential of the setting for the development of numeracy. The theoretical framework adopted is particularly informed by the work of Jean Lave. Of particular interest are the differences between everyday and school mathematical practices. This chapter focuses on differences in the practices between the settings of workplace and school.

Introduction

The focus of this chapter is a module that includes experience in the workplace in a small-scale light engineering context for Year 10 (15 - 16 year old) students. The factory forms one division of a multinational company that specialises in the manufacture of products for the electronics industry. The Application of Number forms a part of the GNVQ scheme, as one of the Core Skills elements and the potential of the setting for the development of numeracy was a particular focus of interest in the study.

The student experience was structured around a series of eight activities, each of which was designed to take place in an afternoon session over the Autumn Term of 1996. The group was composed of twelve students who were deemed to be in need of additional academic support. The decision to offer the opportunity to these students in the first place was taken on the grounds that the traditional academic curriculum was not best serving their needs and interests, although there was an intention to expand such opportunities in the future to a wide group of students. The students were grouped in pairs and each group worked with an adult mentor who was an employee of the factory. The tasks in which the students engaged are outlined later in this chapter. However they did work alongside factory workers in their day to day activities involving the assembly, storage, despatch and quality control procedures of the factory. Several of the tasks, and many of the functions within the factory, related to the process of statistical quality control and these form the focus of this chapter.

Learning mathematically as social practice

In this study I draw on the insights offered by Lave and Wenger's (1991) social practice theory further illustrated in Lave (1988 and 1996). This perspective offers a view of learning as a process of participation in communities of practice, which is at first 'legitimately peripheral' in relation to any new practice but that increases gradually in engagement and complexity. Learning is located in the processes of co-participation, as opposed to within the heads of individuals. The learner acquires the skill to perform by actually engaging in the process, under the conditions of *legitimate peripheral participation* (LPP), to a limited degree and with limited responsibility. Those participating in the community are seen as learners and learning, as such, is distributed among co-participants and is not seen as a one-person act.

With regard to understanding, this is not seen to arise out of the mental operations of a subject on objective structures, rather it is located in the increased access of learners to participating roles in expert performances. Learning can be a feature of various practices and is not seen to be limited to examples of training and apprenticeship. For example, the production of language can be seen as a social and cultural practice. Lave and Wenger's notion of LPP can be seen as a way of engaging and as an interactive process in which the apprentice engages by simultaneously performing in several roles. Learning is seen as a way of being in the social world rather than as simply a way of coming to know about it. Learners are actively engaged not only in the learning contexts but also in the broader social world and learning presupposes engagement without which no learning will occur. Also of relevance is the notion of 'structuring resources' as proposed by Lave (1988) and as discussed by Pozzi, Noss and Hoyles (1997).

Methodology

My approach to this study was that of participant observer and I aimed to attend as many of the factory visits as possible. I worked alongside the students as far as possible although in many situations I was more of an active onlooker and participant in discussion. Data was collected by the use of field notes together with a video camera to capture the detail of the activity. In addition, a single group interview was conducted with the students and semi-structured interviews with four of the staff involved in the process were also carried out, at the end of the programme. Three of the adults were staff mentors together with the Operations Manager who was the driving force behind the initiative with the College.

In deciding on the interview schedule I was particularly influenced by Jean Lave's contribution to the Oxford Seminar in 1996 at which she proposed that the study of learning elsewhere than school offers clearer windows on what

learning is all about. Other work of significant influence has been that of Lerman (1996) who suggests that much greater attention might be given to an awareness of 'the differences between everyday and school mathematical practices and meanings, and between different, mostly workplace out-of-school practices and meanings'. Accordingly I chose to ask about 'differences' and 'similarities' between school and workplace mathematics which led to some quite illuminating responses. I also asked the adult mentors to say how 'good' they judged themselves to be at mathematics which led to some quite rich 'retellings of performance events' (Lave & Wenger, 1991).

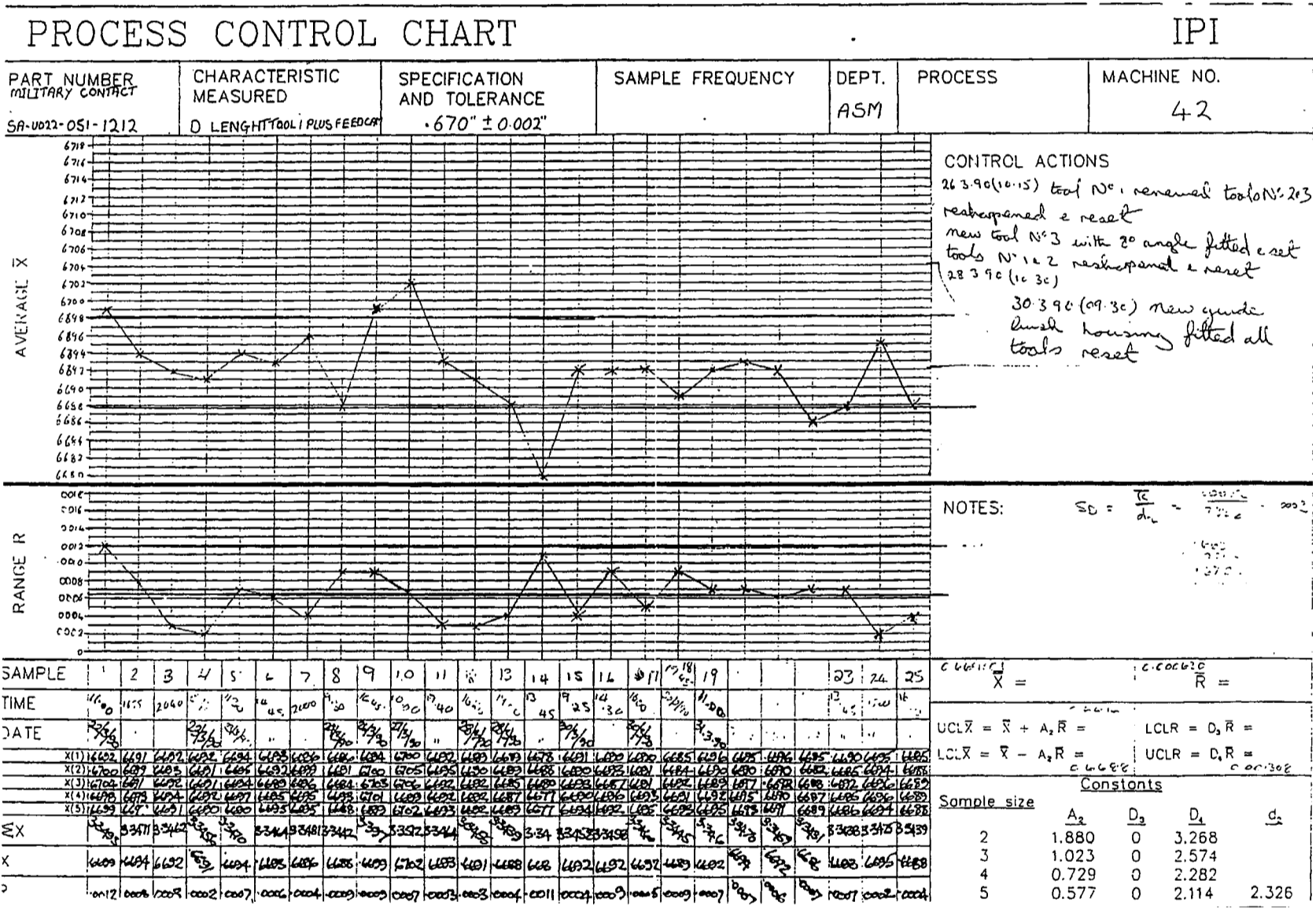
Factory visits

The factory was centred on the production of components for electronic devices such as satellite dishes. These were machined parts that, in the main, were part of an automated production process using high technology computer controlled lathes. In order to monitor the various processes that were underway within the factory, samples of output typically were taken on the start-up of a new process and then at regular intervals after that. The results of this process were designed to allow for the resetting and adjustment of the machines, if necessary, ensuring consistent quality of output and avoiding the chance of producing scrap material as a result of a defective process.

A particularly strong initial impression was the extent to which statistical process control (SPC) was a central feature of the working practice of the operation. As indicated in the introduction, several of the student tasks related to this process which underpinned many of the functions within the factory.

An example of one of the major methods for carrying out this monitoring process is by the use of a Process Control Chart as illustrated in Figure 1.

Figure 1



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The completed chart illustrates how 25 sets of 5 samples were taken over a nine-day period. In each case the sum, mean and range is calculated, and the mean and range are plotted on the corresponding charts above. This chart can be seen as an example of a structuring resource (Lave, 1988 and Pozzi, Noss and Hoyles, 1997).

A second method for monitoring the production process involved the use of devices such as digital verniers and digital micrometers linked to mini-processors. These devices needed to be calibrated initially, based on information from the technical specification of the part, after which a series of measurements would be taken and the mean, range and standard deviation would be automatically computed and printed out.

The mathematical practices on the factory floor

In order to convey some of the mathematical practices of the work in the factory setting I will refer to the words of Jane, who is a 'Leading Hand' on the factory floor. Her role involves organising the workforce in a particular section of the factory. She has been employed at the factory for about ten years:

There are bits of maths in quite a bit of my job - my job actually is inspection and you have to tally works orders up to make sure they are right. You have to make sure all the amounts are correct, so you've got different forms of arithmetic like on the back of a mix order you've got the parts made, parts ... and then you have to carry on parts to the next operator, you have to make all your parts tally all the way through. And it can be very complicated sometimes, cause sometimes you make parts but we don't send them all through if we haven't got a full tube or anything ... but it's tallying everything up so at the end we can have a proper tally.

Especially there's maths especially the SPC in capability studies but at the moment one of the girls downstairs is doing some tests on some parts ... but we are not using the mini processor, we are using the hand-written capability studies and that involves you've got your writing down, measuring ..., reading that off... rounding up and rounding down to ... 3 or 4 figures - then you've got the use of tally charts, you have to read the tally charts, we've got to put it into frequency, percentage, etc. and then it's got to be copied on a graph and then we work out all from this. I can work out all the percentages of possibilities of things going out of control etc....

As I say we've got the use of the vernier, which is, round up or round down, whichever way you want to use it but we mainly use that. Now we've gone off the sheets and gone onto computers a lot. There's a lot less mentally, you've got to put in your numbers correctly, otherwise you end up with something totally out of control which happens sometimes.

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There's also reading of your graphs, reading from information, using your scales, that also involves maths mostly to a smaller degree; cause you could be measuring the outer parts, then there's a way you can use your scales to measure them more accurately. So you have keep on there's a way of doing it where you are just putting a few parts, putting a few more and press a few buttons and it 11 come out

This account confirmed the observations in the factory of a wide range of mathematical practices that can be related to mathematics in the National Curriculum. With regard to the 'Using and Applying of Mathematics' component of the National Curriculum, there was much evidence of using and applying mathematics in practical tasks and real life problems. 'Making and monitoring decisions to solve problems' involved reviewing progress and checking and evaluating solutions. In relation to 'communicating mathematically', there was a need to understand mathematical language and notation, use mathematical forms of communication, including diagrams, tables, graphs and computer print-outs, interpret mathematical presentation in a variety of forms and examine and evaluate these critically. With regard to 'number', there was a need to understand and use relationships between numbers, understand and calculate averages and develop methods of computation including calculators and calculating devices. In relation to 'algebra', there was a need to understand and use formulae and expressions, and to interpret and evaluate these in real life situations using computers and calculators as necessary. Finally with regard to 'handling data' there was the need to process and interpret data, interpret a wide range of graphs and diagrams and to evaluate results critically. Some of the complexity is conveyed well by Jane below in her description of her day to day work.

Adult mentors 'retellings of (mathematical) performance'

The adult mentors were Linda, Janet and Jane. Each was interviewed at the end of the project using a semi-structured interview approach. One general observation that could be made was that despite being very capable in their workplace roles, all three adults exhibited a low level of confidence with abstract mathematics. Also they did not see the relevance of such mathematics. In fact Jane, who was the most able mathematically, had struggled to pass her GCSE in mathematics at the third attempt and yet recalled the ease and enjoyment with which she had worked with statistical ideas in her A Level geography course where it was related to people:

I think it's like people relate to ... like ... people where it's put into relation to people or things but where it's figures it tends to overload me sometimes I think ... But A level geography it was that side I enjoyed that far more than the physical side of geography, where it was related to people, cities etc. Why people do this and why they do that...

Janet was less confident in her mathematical ability:

But yes you do have to be pretty good at maths, it isn't my strong point, I've got a calculator.

In reflecting upon her experience of school mathematics, she emphasised the idea of doing 'exercises':

I think in school you just like getting your exercises right, it weren't like finding things like in a drawing like we do. I've never come across that till I came to work here. You didn't actually measure anything... I prefer it as it is now.

Linda also saw herself as not being very good at mathematics in school, although she felt that her mathematical ability had improved since working at the factory. Also, she saw the use of a calculator as a basic mathematical skill and not as a sign of her inadequacy, as it seemed in the case of Janet:

I'm not numerical, I've never done maths, I wasn't very good at it at school, I've got better since I've come here. It's got a lot easier since I've been in stores, than whatever I did at school so I think it's good.

When asked how she coped with arithmetic, she emphasised the need to use the calculator:

As long as I've got a calculator there, which you have to have because your customer demands that he has that quantity and because every single thing is logged on to a computer, if you miss one piece you know about it, do you know what I mean it's so very spot on, immaculate and everything that you've got to spot on ... I use a calculator but you never did when we were at school so you've got to learn how to use a calculator. I mean some kids haven't got a clue how to use a calculator so I think you should be taught how to use one properly.

Discussion

In analysing the responses of the adult mentors, a particularly relevant aspect of social practice theory is Lave and Wenger's thinking about the notion of 'engagement' and in particular the proposal that

learners are actively engaged not only in the learning contexts but also in the broader social world and learning presupposes engagement without which no learning will occur, (my underlining)

In reflecting upon her experience, Jane distinguishes between her enjoyment of the mathematics in her A Level geography when it was about 'people' in

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contrast to being just about 'figures', which might be seen as abstract mathematics which 'tends to overload me'. However she proceeds to emphasise *purpose* also i.e. 'Why people do this and why they do that'. This sense of purpose reflects Lave and Wenger's notion of 'engagement' and is consistent with activity in the strong sense of the term as discussed by Crawford (1996). Lave highlights how activity denotes personal (or group) involvement, intent and commitment that are not reflected in the usual meanings of the word in English. She draws attention to the fact that Vygotsky (1962) wrote about activity in general terms to describe the personal and voluntary engagement of people in context - the ways in which they subjectively perceive their needs and the possibilities of a situation and choose actions to reach personally meaningful goals. In her recollections, Janet seems to emphasise *the lack of purpose* in school mathematics i.e. it is 'just about getting answers right' (in school) and not 'like finding things out like in a drawing like we do'. She emphasises that she had 'never come across that' (sense of purpose) until she 'came to work here'. A further relevant aspect of Janet's view is the way in which she sees the calculator as a tool that is taken for granted. Linda also emphasises the use of a calculator. However she stresses the need to use a calculator for a purpose i.e. 'because your customer demands that he has that quantity' and 'it's so very spot on, immaculate ...'

A number of the issues arising from the interviews with the adult mentors were reflected in the feedback from the students. This feedback was obtained from a single semi-structured group interview with all the students at the end of the project. When asked about what was different in the factory setting from the mathematics done in school, an immediate response was 'it's rubbish at school' and when asked to give reasons the responses were that 'it's boring' and 'it's harder'. These responses drew general agreement from the group as a whole. When asked why it is harder, the immediate response was that 'you don't get any homework'. However when pressed to say more the response was that 'because they're testing you in school'. This comment was echoed strongly by others in the group. When asked whether they were being tested in the factory, the response was:

Not really ... they weren't testing you were they? They were showing you how to do things.

When pressed to say more about why it is boring in school, the response was

You get it all the time. It's just boring. It's not practical ... You just sit down don't you?

In recalling what the students were saying there was a strong sense of conviction and general agreement about how they found mathematics to be 'boring', 'not practical' and just about 'testing'. The suggestion that school mathematics is not practical is consistent with the responses from the adults

i.e. mathematics without a purpose. The expression of boredom conveys that overwhelming sense of waste when one is *not engaged* with something and yet unable to escape from it. However the comment that 'it's harder ... because they're testing you in school' also conveys some of the impact of the National Curriculum upon this particular group of students. The 'testing and examination' culture associated with accountability and external control as described by Gipps (1994) was very apparent through these comments. She contrasts this culture with that of an assessment culture associated with teaching, learning and formative assessment which seems to have been far more evident in the factory setting than in that of the school for these students.

One of the reasons for developing the link with the school by the factory was the poor take up of opportunities to work in manufacturing within the local area. A deep resistance was perceived, especially on the part of local parents. However at the end of the programme two of the students in the group were very interested in the possibility of taking up apprenticeships at the factory and were thought to be very suitable candidates, despite the fact that they were not seen to be succeeding in school. This is indicative of a wider problem within the education system as a whole that is not being addressed by the current preoccupation with performance indicators, testing, targets and school league tables.

The accounts from the adults of their school mathematics conveyed a generally low level of confidence and yet in the workplace they were using mathematical skills appropriately, effectively and with confidence. This raises serious questions about what the school system is achieving in terms of a mathematics curriculum 'for all'. A number of echoes could be found in the comments from the adults with what the students had to say about their current experience of school. For example, the relationship with the 'real world' seemed to be powerfully engaging, as did the idea of doing mathematics with a purpose in a practical setting. Given the current debate about the role of the calculator, it was especially interesting to note Linda's comments on her use of the calculator as a tool and also on her view of the need to teach students how to use such tools effectively.

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Section Two

Mathematics at Work

Chapter 6

A TALE OF TWO SCHOOLS: ALTERNATIVE TEACHING APPROACHES AND SITUATED LEARNING

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This chapter provides an overview of the results from ethnographic case studies of students in two schools. The schools used radically different approaches to the teaching of mathematics; one school using textbooks and whole-class teaching, the other using open-ended projects. Approximately 300 students were observed, assessed and interviewed over a three-year period. At the end of that time, the students at the textbook school had developed limited forms of 'classroom knowledge' that they could not, or would not, use in novel situations. They believed that the demands of the classroom and the 'real world' were irreconcilably different and this rendered much of their school learning of mathematics ineffective. The students at the project-based school were more effective in their use of mathematics inside and outside of school, partly because of their views about the nature of knowledge-use. The differences between the learning of the students in the two schools are considered in relation to notions of situated cognition.

Introduction

The idea that students do not use the knowledge they gain in classrooms when they are outside of school, because the context, situation and goals that are formed in relation to non-school situations are different, has much common sense appeal and has been embraced by many in education circles. Such notions provide direct opposition to the theories of 'learning transfer' that underpin many of the practices in educational institutions. Learning transfer theories are based upon the idea that knowledge can be taken from one situation to another when information is learned, links with a new situation are recognised and information is successfully retrieved from memory. Opponents of learning transfer argue that such theories are simplistic, based upon insubstantial evidence and derived from fallacious assumptions which are mainly functionalist in origin (Lave 1988). The functionalist influence upon these theories displays itself in the conception of knowledge as a set of tools, stored in the memory to be taken out and used whenever necessary. These tools are seen as discrete entities, impervious to processes of socialisation or to the environment or context in which they are required, they merely form part of a pool of information that is transmitted from one generation to the next. Jean Lave has been one of the leading voices in the opposition to notions of transfer, pointing out that theories that separate cognition from the social world are no longer tenable (1988, 1991, 1993). Such theories, she suggests, represent knowledge as a series of

'coherent islands whose boundaries and internal structures exist, putatively, independently of individuals, except that they have more or less of it' (1988 p43). Lave (1993) insists that when knowledge is brought to bear upon a situation it is always a product of the people, their activities, their interests and goals and the ways that these relate to the situation they are in. Transfer theories are redundant in this perspective, not because individuals do not make use of knowledge gained in one setting in another, but because the knowledge that is used in a new setting is always created in and for that setting. As Hutchins has proposed, 'the properties of the *interaction* between individual minds and artifacts of the world' (1993, p62) (*italics mine*), are at the essence of human performance. Notions of transfer give no indication of this changing, relational nature of cognition.

The implications of this debate for schooling practices cannot be overstated and I would like in this chapter to relate the perspective of situated cognition to events within two schools. There isn't the space to report upon the case studies I conducted in the two schools in any depth, but details of the two schools, the general issues that emerged from them and the observations, assessments and interviews I conducted are provided elsewhere (Boaler, 1997a, b,c; 1998).

The Research Study

In order to monitor and consider the mathematical knowledge and understanding that students develop in school I conducted three-year case studies of a cohort of students in two schools. The students were matched at the start of the research period in terms of sex, social class, gender and mathematical attainment. My enquiry comprised a longitudinal study of a 'year group' of students in each school as they went from year 9 (age 13) to year 11 (age 16). I chose the two schools because the intakes of students were very similar, but the schools offered completely different approaches to the teaching of mathematics. At 'Amber Hill' school the students were taught using a traditional, textbook approach. At 'Phoenix Park' school the students worked on open-ended projects at all times. When students needed to learn about a new area of mathematics at Phoenix Park, the teachers would teach it to them within the context of the situation in which they were working. This feature of Phoenix Park's practice was shared with some classroom 'apprenticeship' models of teaching, designed to acknowledge the situated nature of learning (Cognition and Technology Group at Vanderbilt, 1990).

Amber Hill was a traditional school and the students were disciplined and hard working. In lessons the students were passive, but conscientious, and they strove to learn all of the different methods and procedures they were introduced to. During the three years I spent at Amber Hill, observing approximately 100 lessons and performing a variety of assessments, I found that the learning of the students was remarkably ineffective, and although

students could work through their textbook exercises with ease, they found it difficult applying the methods they learned to anything but textbook situations. This was demonstrated on a range of assessment occasions when the students became confused when they were not told exactly what method to use. The students themselves became aware of these limitations when they took their mock GCSE examinations. Until that time, they had thought that they would be successful in mathematics if they learned all the rules and formulae they were introduced to in their lessons. In the examination the students found that this was not the case:

A: It's stupid really 'cause when you're in the lesson, when you're doing work - even when it's hard - you get the odd one or two wrong, but most of them you get right and you think well when I go into the exam I'm gonna get most of them right, 'cause you get all your chapters right But you don't (Alan, AH, year 11, set 3)

In interviews at Amber Hill (n = 40) the students reported that they saw no use for the mathematical methods they learned in class in 'real world' situations. This was because they regarded the mathematical demands of the school and the 'real world' as inherently and fundamentally different:

JB: When you use maths outside of school, does it feel like when you do maths in school or...

K: No, it's different

S: No way, it's totally different (Keith and Simon, Amber Hill, year 11, set 7)

G: I use my own methods.

JB: Why is that do you think?

G: 'Cause when we're out of school yeah, we think, when we're out of school it's social, you're not like in school, it tends to be social, so it would be like too much change to refer back to here. (George, Amber Hill, year 10, set 3)

At Amber Hill the students were remarkably consistent in their mathematical behaviour. The students were generally motivated and they worked hard in lessons, but in applied settings, examination questions and 'real world' situations they found that they were unable to use the procedures they had learned. This was because they were rarely able to change or adapt the methods they had learned to fit the demands of new situations, they did not believe that it was appropriate to 'think' in mathematical situations (only reproduce standard procedures) and they perceived school mathematics to be irrelevant for anything other than textbook questions.

At Phoenix Park the situation was very different. In a range of applied assessments the students attained significantly higher grades than the students at Amber Hill. In the GCSE examination the Phoenix Park students

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attained similar proportions of A-C grades, but significantly more A-G passes than the students at Amber Hill. This difference was despite the fact that the students were matched in terms of attainment at the start of the study, the Amber Hill students were more 'disciplined' and spent more time-on-task (Peterson & Swing, 1982) and the Amber Hill approach was much more 'examination-oriented' than the open, project-based approach of Phoenix Park. In interviews the students at Phoenix Park reported that they saw very little difference between the mathematics of the 'real world' and the classroom and so, they reported, they made use of school-learned mathematics in real situations.

I will now explore the idea that the differences between the actions and perceptions of the students at the two schools were due to the situated nature of learning.

Situated Learning

The Amber Hill model of mathematics teaching, predominant in the majority of UK schools (OFSTED, 1994) was based upon notions of transfer. The assumption behind Amber Hill's textbook, transmission approach was that if students were taught abstract, generalised mathematical principles they would be able to take these and use them in a range of different situations:

JB: What do you normally do in a maths lesson?

J: Well sir usually goes over the work we have to do before we do it, so he'll write it on the board what we have got to do and explain the questions and that and the rules, the basics of what we have to do in the work and then he'll tell us to get on with it (John, AH, year 10, set 1)

At Phoenix Park the students were encouraged to learn mathematics through 'continuing authentic activity' (Brown, Collins & Duguid, 1989). The students tried to solve mathematical problems, learning about new methods along the way, and were essentially 'apprenticed' into mathematical use. The impact of this approach upon the students was that it encouraged them to think about the situations they were in and to adapt and change mathematical methods they knew, to fit the demands of different situations. If they encountered a problem they had not met before, they were generally prepared to try and solve it. This propensity undoubtedly helped them in the GCSE examination:

JB: Did you feel in your exam that there were things you hadn't done before?

A: Well, sometimes I suppose they put it in a way which throws you, but if there's stuff I actually haven't done before I'll try and make as much sense of it as I can, try and understand it and answer it as best as I can, and if it's wrong, it's wrong. (Angus, PP, year 11)

Part of the Phoenix Park students' success appeared to derive from the relational, situated model of learning that they, themselves believed in. When the students encountered mathematical problems they did not try and 'transfer' set pieces of mathematics, they used their past experiences to inform their thinking. When the students talked about their use of mathematics, in interview, they talked in terms that were remarkably consistent with the perspective of Lave. Consider, for example, the following two extracts:

JB: Is there a lot to remember in maths?

S: There's a lot to learn, but then you need to know how to understand it and once you can do that, you can learn a lot

P: It's not sort of learning is it?, it's learning how to do things.

S: Yes, you don't need to learn facts, in the beginning of the maths paper they give you all the equations and facts you need to know. (Philip & Simon, PP, year 11)

JB: How long do you think you can remember work after you've done it?

G: Well I have an idea a long time after and I could probably go on from that, I wouldn't remember exactly how I done it, but I'd have an idea what to do. (Gary, PP, year 11)

Both of these extracts seem important to consider. In the first, Philip and Simon concur with Lave's claim (1996) that notions of knowing should be replaced with notions of doing, in order to acknowledge the relational nature of cognition in practice, as illustrated by the distinction drawn out by Philip: 'It's not sort of learning is it?, it's learning how to do things'. This comment also highlights the difference between the Amber Hill and Phoenix Park approaches. At Amber Hill teachers tried to give the students knowledge, at Phoenix Park the students 'learned how to do things'. This distinction led to differences in the mathematical beliefs of the students – at Amber Hill the students thought that they needed to remember a vast number of rules and procedure, at Phoenix Park the students thought that mathematics involved working things out. These beliefs meant that in mathematical situations the Phoenix Park students were not inhibited in the way that the Amber Hill students were. They were not struggling to remember set procedures, nor search for cues (Boaler, 1996,1998) which may indicate the procedures to use. They were free to consider the different questions and make sense of them, reflecting on their past experiences.

Gary's comment is also important because he appears to suggest, quite explicitly, that he does not 'transfer' pieces of knowledge, rather, he creates new ideas in relation to the situations he is in. Gary adds support to a relational view of knowing, because he dismissed the view that knowledge existed in his head ('I wouldn't remember exactly how I done it') and stated that his knowledge would only be *informed* by previously held ideas, he would 'go on from that' and form ideas of what he had to do in different

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situations. Part of the Phoenix Park students' success seemed to derive from the beliefs that the students themselves held about the situated nature of learning - they knew that they only needed to remember an idea and move on from that. The Amber Hill students tried to remember set pieces of knowledge and apply them, which often meant selecting from complex sets of algorithms and procedures they had been taught. The Phoenix Park students did not even attempt to do this, they tried to form or create new knowledge, informed by their previously held ideas.

These differences were not the result of good and bad, or popular and unpopular teachers at the two schools, they were related to the different *models* of teaching employed and the fact that the Phoenix Park students were *apprenticed* into a system of mathematical thought and work:

L: Yeah when we did percentages and that, we sort of worked them out as though we were out of school using them.

V: And most of the activities we did you could use.

L: Yeah most of the activities you'd use - not the actual same things as the activities, but things you could use them in.

JB: If you were in a situation outside of school and you needed to use some maths do you think you would remember back to things you have learned here or do you think you would use your own methods?

L: Um, sometimes I know I have changed methods to make it easier for me - if you find it easier the way you learned it then you keep the same, whatever's easiest (Vicky & Lindsey, PP, year 11)

These students again support a situated view of learning (Lave, 1993), because they describe the way in which they developed meaning in interaction with different settings. Lindsey said that she would use mathematics 'not the actual same things as the activities, but things you could use them in', she would adapt and transform what she had learned to fit new situations. Later in the interview she said:

L: Well if you find a rule or a method, you try and adapt it to other things, when we found this rule that worked with the circles we started to work out the percentages and then adapted it, so we just took it further and took different steps and tried to adapt it to new situations. (Lindsey, PP, year 11)

The analysis offered by Lindsey in this extract is very important, for it was this willingness to adapt and change methods to fit new situations which seemed to underlie the students' confidence in their use of mathematics in 'real world' situations. Indeed many of the students' descriptions suggest that they had learned mathematics in a way that transcended the boundaries (Lave, 1996) that generally exist between the classroom and the 'real world'.

Lave (1996) asserts that students do not use mathematics learned in one situation in another situation because the two situations represent different 'communities of practice'. The students relate to them differently and form different ideas in relation to the two settings. This analysis is similar to one offered by Bernstein (1971) in which he suggests that educational knowledge is 'uncommonsense knowledge' (p58) and that children are socialised early in their lives into knowledge frames which discourage connections with everyday realities. Within school the Phoenix Park students did not view mathematics as a formalised and abstract entity that was only useful for school mathematics problems. They had not constructed boundaries (Siskin, 1994) around their school mathematical understandings in the way that the Amber Hill students had. At Amber Hill the students developed a narrow view of mathematics that they regarded as useful only within classroom, textbook situations. The students regarded the school mathematics classroom as one 'community of practice' (Lave, 1993, 1996) and other places, even the school examination hall, as different communities of practice.

Lave (1996) claims that learning would be enhanced if we were to consider and understand how barriers are generated that make individuals view the worlds of school and the rest of their lives as different communities of practice. At Amber Hill there were strong institutional barriers that separated the students' experiences of school from their experiences of the rest of the world. Many of these barriers were constituents of Bernstein's visible pedagogy (Bernstein, 1975). General school rules and practices such as school uniform, timetables, discipline and order contributed to these as well as the esoteric mathematical practices of formalisation and rule following. At Phoenix Park the barriers between school and the real world were less distinct: there were no bells at the school, students did not wear uniform, the teachers rarely gave them orders, they could make choices about the nature and organisation of their work and whether they worked or not, mathematics was not presented as a formalised, algorithmic subject and the mathematics classroom was a social arena. The communities of practice making up school and the real world were not inherently different. The importance of the students' perceptions about the formality of the mathematics classroom at Amber Hill was shown very clearly by George's comment given earlier:

G: 'Cause when we're out of school yeah, we think, when we're out of school it's social, you're not like in school, it tends to be social, so it would be like too much change to refer back to here.

In the mathematics classrooms of Phoenix Park talk, discussion and negotiation were intrinsic features of the students' work. At Amber Hill the students were allowed to talk to their partners as they worked, but the students clearly did not view the mathematics classroom as a social arena. This was important, partly because the Amber Hill students experienced less opportunity to derive meaning through a discussion of mathematical

concepts and partly because this contributed towards the students' perceptions of difference. George gave a clear indication that he regarded the classroom and the rest of the world as different communities of practice and this meant that the mathematics he learned in school was of no use to him outside of school.

Conclusion

The results of this study may be considered both in terms of emergent learning theories and, importantly, school policy and practice. For example, when the Amber Hill students could not, or would not, use school mathematics in non-school settings this was not because they had learned it in a confusing way, but because their goals were different, their interpretations of experience were different and they located themselves and their knowledge-use *in terms of* the social world. The students' reflections lend direct support to Lave's relational knowledge, in particular, the interdependency of person, activity, knowledge and setting (Lave, 1993). The results have also shown that attempts to impart knowledge to students, such as those which underpin the educational policies of the 'New Right' (Ball, 1993, p195) and, increasingly, New Labour, are less helpful than classroom environments in which students are enculturated and apprenticed into a system of knowing, thinking and doing. But such environments are scarce in the centralised, uniformity-driven political climate of the 1990's; even Phoenix Park has now abandoned open and apprenticeship models of teaching, in response to mounting pressures to adopt the traditional, textbook approach that encapsulates the ideology of transfer.

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Chapter 7

PARTICIPATING IN LEARNING MATHEMATICS THROUGH SHARED LOCAL PRACTICES IN CLASSROOMS

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In this chapter a theoretical perspective is developed from which to identify and describe local communities of practice which are useful in thinking about mathematics teaching and learning. This perspective is exemplified with descriptions of individual mathematics lessons and brief consideration is given to the implications of its application to wider notions of schooling.

Introduction

Theories of situated cognition provide us with tools for analysing apprenticeship models of learning (Lave 1988, 1993, 1996, Lave and Wenger, 1991). What is more they suggest that learning only takes place within communities of practice. Seeing schools and classrooms as learning communities has encouraged some writers to attempt to superimpose the apprenticeship model onto school learning. It is not clear whether schools fit in at all with an apprenticeship model of learning, and if they do it is not clear how (see Lerman and Adler, this volume). Learning, for Lave, has no necessary connection with deliberate teaching; the learning to which she refers is directly related to the practices of the community whereas the knowledge taught in school bears little relation to the practices and, indeed, the actual function of the school as a community.

There have also been useful applications and adaptations of elements of this theoretical perspective. Boaler for example (1997, 1998) uses Lave's ideas about learning transfer to develop a convincing critique of traditional, transmission models of mathematics teaching and learning and assessment in particular. Theories of situated cognition have also been deeply influential on the ideas of other key figures engaged in mathematics education research; Noss and Hoyles' idea of 'situated abstraction' (1996), mathematical abstraction which is specific to the working domain, for example, could be understood in this context.

We will return briefly in the conclusion to the important task of somehow coming to see schooling in general in terms of practice. In this chapter we want to focus more narrowly on the development of a language and vocabulary which enables us to describe what will be called local shared practices, or local communities of practice. So, whilst we will put off the solution of the larger

problem of whether theories of situated cognition apply to schooling, we will nevertheless attempt to make some useful statements about teaching and learning.

Community of practice

Terms like 'communities of practice' seem very much a part of current discourse and, perhaps for this reason, and in spite of Lave's writing, their meanings may not always be clear. For this reason, too, we want to make clear what we understand by the term. A community of practice, in the sense in which we use it here, must have certain necessary features¹:

1. participants, through their participation in the practice, create and find their identity within that practice (and so continue the process of creating and finding their more public identity);
2. there has to be some social structure which allows participants to be positioned on an apprentice/ master² scale;
3. the community has a purpose;
4. there are shared ways of behaving, language, habits, values, and tool-use;
5. the practice is constituted by the participants;
6. all participants see themselves as engaged essentially in the same activity.

The first four features could be interpreted to be true of schools in some respects; it is less obvious that the last two could be seen in (the formal instruction side of) schools. In most mathematics lessons the teacher is not engaged in learning mathematics, although both teacher and pupils could be, and sometimes are, engaged in doing mathematics. Whilst we might say that pupils, as well as teachers, together constitute the practice within all classrooms, pupils' participation is often passive, and therefore if they can be said to constitute the practice this would only be through their acquiescence.

Local (communities of) practice.

We believe that a way forward might be to describe schools in terms of multiple intersections of practices and trajectories. Within schools, however, we believe that it is possible to talk sensibly of local communities of practice (Lave 1993). Such communities may be local in terms of time as well as space: they are local in terms of people's lives; in terms of the normal practices of the school and

¹Listed in this order for convenience of reference only.

²We use these terms because we want to emphasise the social nature of such judgements. The expert/novice distinction has the attraction of gender neutrality, but it suggests a kind of cognitive-psychological activity and we are not focusing on that here.

classrooms; in terms of the membership of the practice; they might 'appear' in a classroom only for a lesson and much time might elapse before they are reconstituted (although it may be possible to detect the subtle effects of the echo that remains after their passing in the trace of learners' trajectories or the development of other practices). Apart from these spatial and temporal constraints, local communities of practice (LCP) display all the elements we have listed.

We find the construct of local community of practice to be both useful and usable: it is possible to identify LCPs through observation; setting out to initiate the creation of LCPs would be, we suggest, a support to planning for the effective mathematics learning of students. The model should be accessible to colleagues, both fellow teachers and beginning teachers, and so provide a focused mechanism for the study of pedagogy. We could, for example, ask our students how they proposed to initiate the construction of local communities of practice within their lessons.

We will now give two examples of local communities of (mathematical) practice. We provide a third example to show where we detect no such local practice.

Example 1: Exploring the graphical calculator

Each member of a class of 13 year-old girls was given a graphical calculator to use as her own. These calculators (in 1996) were very powerful³, but the feature that was central to the establishment of the practice described here was that any machine could be connected to an overhead projector display screen.

For their first activity using the calculators the students, were asked to work individually and in groups to explore the machines. No explicit mathematical agenda was set, but students were asked to respond to these questions:

- How is the calculator similar to other calculators you have used?
- How is it different?
- What is there, if anything, which surprises you?

Students did their initial exploration by themselves at home. The next day they presented their personal responses, observations and ideas to the others in their group and together the group planned a joint presentation to be made to the whole class in the following lesson. In preparatory discussion with the teachers (the researcher acted as a teacher) the focus was on the skills needed to be able to

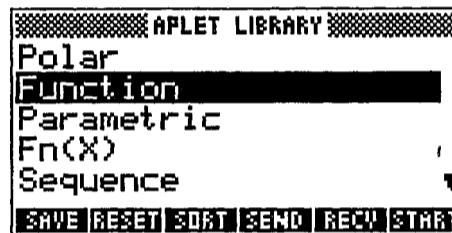
³Hewlett Packard HP38G

work effectively as a group, in particular skills such as listening, helping, asking useful or helpful questions.

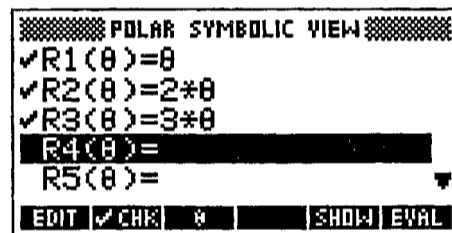
Sara's group decided that they would include her report on her exploration of polar co-ordinates.

Sara refers to her mathematical activity using the situated symbolism provided by the calculator; she does not otherwise label her mathematical activity. This may be an important aspect both of calculator use and of setting up local practices, but we do not go into that in detail here. Here is part of Sara's presentation to the whole class:

Sara starts by saying she has 'just messed around'. She turns on her machine and displays something like this:



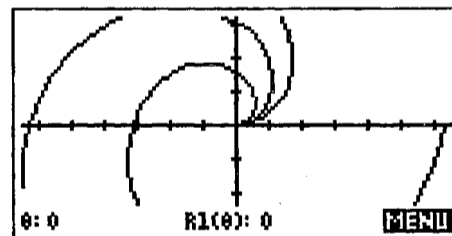
It represents a set of possible domains in which she might explore.



Sara selects *Polar* and so displays a set of polar functions she has defined beforehand:

She says:

After you put in any, you know, you want...then you just press PLOT...and, I just love this one...it comes out so lovely...



Class: Ooh,wow...

someone in the class says: My one doesn't work.

Sara: Your scales aren't the same as mine...must be the axes...I've actually zoomed out.

Teacher: How could Sara show us what the axes are?

Sara: All I did on mine was just zoom out.

Through a combination of the setting and the technology Sara, the others in her group and the rest of the class together validated the investigation of polar equations as an appropriate mathematical activity - albeit one that was outside the curriculum. Other groups of children were similarly willing to explore the calculators and explain what they had found. One group, for example, showed how they had used the calculators to draw pictures with a 'pencil' and to write notes; another group showed how to send messages from one machine to another; yet another group explained that they had been exploring graphs of function that made use of the SIN key. This willingness to explore, explain, listen was taken to be a defining characteristic of this local community of practice. It is possible that Sara's central role in its establishment may prove to have been pivotal in her developing identity both within and beyond the classroom. She had always been seen as 'good at mathematics' - she was now seen to be becoming a master of this calculator-mediated mathematics.

Example 2: Silent fractions, smiley faces

At the beginning of this lesson the teacher welcomes her class in her usual friendly manner, but she says nothing, not a word. After a while the children, boys and girls of about 11 years old, begin to settle in their places but they are puzzled....

writes on the board..

$$\frac{11}{16} + \frac{5}{16}$$

and a little later....

$$\frac{3}{11} + \frac{8}{11}$$

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Still she does not talk. There is, perhaps, some embarrassment in the class. There are one or two titters, but not much else.

The teacher faces the class. They know her well enough to recognise from her stance that she wants them to begin to join in. They begin to watch more carefully.

The teacher offers help, but no utterance...

$$\frac{1}{8} + \frac{5}{8}$$



She adds to the fraction additions she wrote earlier...

$$\frac{11}{16} + \frac{5}{16}$$



$$\frac{3}{11} + \frac{8}{11}$$



Minesh walks to the front of the class. Without talking the teacher offers him the chalk. Without talking he writes:
and adds his own smiley face...

$$\frac{1}{3} + \frac{2}{3}$$



Before long more children begin to step forward. They add to what is on the board. Participants either follow the current rules or they introduce a new one...

$$\frac{2}{8} + \frac{3}{4}$$



$$\frac{2}{9} + \frac{3}{9} + \frac{4}{9}$$



a little while later, Susan writes this:

$$\frac{2}{7} + \frac{5}{7}$$

but then adds a sad face:



Then she writes this:

$$\frac{2}{14} + \frac{5}{14}$$

and Stephen adds this:



The lesson ends as it began. The teacher smiles at the children, saying nothing as they leave for the school playground.

Of course, the whole lesson did not pass in total silence. Children could be heard, in varying degrees of whisper, to ask each other what was going on, but their understanding - of the mathematics and of the 'rules of the game' - both was constituted by and constituted the unfolding practice of the class. Silence was an essential feature of this practice, contributing to a setting in which students could be seen as masters as much for their social mastery of the rules within that classroom - there and then - as for their more widely recognised mathematical mastery. The local practice, then, was by itself sufficient to allow students to constitute themselves in some sense as effective mathematicians

Example 3: Four 4s

A class of 11 year-old girls and boys enter the classroom and settle themselves into their seats. The children have just begun their studies at their new secondary school.

The teacher gives the children an example to introduce the task he wants them to do. He explains that you can use four 4s to make the number 9:

$$4 / 4 + 4 + 4$$

He responds during his explanation to questions that some children ask.

Linda: Why isn't that the same as $4 / 12$?

Teacher: (words to the effect that) You do the dividing first to get the same as

$$1 + 4 + 4$$

A few more examples follow and the children then work on these by themselves.

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The teacher sets the children their homework. He asks them to show how to construct as many as they can (he does not say 'all') of the numbers from 1 to 25 using exactly four 4s. The children leave.

At home, Evelyn has completed as much of the homework as she can. She can't yet do 21 and 25 and she wants to do them. She telephones her uncle. They talk for a long time on the 'phone. He helps here with ideas of indices and 4^0 , but suggests to Evelyn that, perhaps, she should only make use of these ideas if she can explain them back to him. Evelyn explains convincingly.

Evelyn completes the homework and hands it in.

About two weeks later, Evelyn and her uncle are talking over a meal. He asks how the homework went. Evelyn explains that the teacher took the books in but didn't get a chance to mark them yet.

About three weeks later, Evelyn's book is collected again. The work is marked and returned. She reads that she has done well. The class has moved on to another topic by now.

From the point of view of the teacher, this third example might appear to be two lessons and a homework. He might well have thought (though not in these terms) that the children's responses came out of the practice that had been established in his classroom. He probably did not see the children's work as a product of the multiplicity of practices whose intersection only he observed in his classroom. Some of his children were participants in practices which included not only the telephoning of mathematically-inclined uncles, but also the expectation that this was an appropriate and natural thing to do. To be sure the success of all lessons must depend to some extent on such multiple participation. We think that this third example is typical of many lessons whose success effectively depends wholly on what children bring with them: no local shared practice of mathematics had been initiated.

Discussion

As we have said, we think that there is a distinction between lessons we choose to call local communities of practice (the practice of doing mathematics) and others where we make no such identification. It is helpful to think of any classroom as an intersection of a multiplicity of practices and trajectories. For example, a classroom may contain pupils whose main current focus is the

football match after school, a row with a friend, their personal history of confusion in mathematics lessons, the mental arithmetic skills required to become a traditional greengrocer, an exciting insight shared with an uncle about last night's homework, or intense worry about an examination result. This is the reality of the situation; it can not be other; the teacher can not believe it to be other. It is a tough task for a teacher to create situations in which all of these learners with their distinct identities developing within multiple practices come to focus on the same issue.

In our first two examples we want to say that, from this rich layering of practices and becomings, local practices emerged which were defined by and required the active participation of those who together constituted those practices; within such practices there is, by definition, a strong social pull on all - including the more peripheral - to participate. In the third example, whilst there may have been pockets of active participation, we want to say that the learning which happened was much more a product of the complex identities in practice that teachers and learners brought with them when they stepped into the classroom than anything that happened to them once they were there.

Local communities of practice can therefore be said to be at least an indicator of effective teaching. For consider the learning that takes place outside of such a practice; this will owe much (everything?) to those larger practices which together constitute the learner's identity - as scholar, perhaps, as a good student of average ability, as classroom clown, as truant; we suspect, too, that even the best teachers in effect rely on their students' unarticulated, unrecognised participation in those other practices for whatever success they might achieve.

Telos

The local communities of practice we have described support a common direction of learning which, from the perspective described here, is a defining feature of those communities. Lave and Packer (Lave 1996) have found it useful to emphasise such direction - telos - as an essential stipulation of any theory of learning. In our reading we understand telos to refer to the way that an individual becomes what they are going to be within a community of practice. The learning they do is both a determinant of this direction and in part determined by the complex paths which students have taken to get where they are. Thus people can appear, superficially, to be learning the same thing but the knowledge they gain, and the effect it has on them, can be very different. For instance, several pupils can learn that $3/16 + 13/16 = 1$: for some this may be obvious and uninteresting; for others it may be an important realisation; for others again it may provide a moment when they begin to feel powerful

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mathematically, and begin to see themselves as valuable participants in the lesson, classroom, or community of practice.

In a broad sense telos is an unfulfilled potential to move or change in many different ways; telos could be conceptualised as a set of constraints in some sense inherent in situations and in the individual's predispositions to respond to situations as she does. Whilst at school, though not necessarily because of school, learners are preparing to become the socially functioning beings they are going to be as well as, we hope, productive adult members of society; their experiences at school are also mediated by the images of themselves that they, as learners, bring with them.

The image of telos that we are working with here is complex. That a learner is located in the same time and space as a LCP is no guarantee of her participation in that practice. But, from the perspective that all cognition is situated, *all* learners are seen as participating in a set of practices; for the 'non-participant' these practices will just be all the other practices which that learner in some sense, 'brings with her' into the classroom. So, we conceive of the telos of an individual learner as an aggregation, an atlas, of the smaller 'becomings' to be identified with that learner's participation in a multiplicity of communities of practice, local and not so local, some of which are locatable within school classrooms. A useful, though inadequate image might be the smooth convolutions of the silken cloth whose structure is seen, on closer inspection, to depend upon the relationship between the myriad threads; the warp and the weft.

Some aspects of the perspective we propose are not new! They can be identified in the movement in the 1970's to educate 'the whole child' (Wilson, 1971) and, of course, in the writings of John Dewey (1916). Lave's contribution is distinctive in that it looks at the learner's experience, rather than the teacher's view, school organisation or curriculum.

From the point of view of the learner in a mathematics classroom, for three times a week, 38 weeks a year, it is seldom clear how one's experiences affect the process of becoming the person one is going to be; possible exceptions are those few who see themselves as joining a community of mathematicians. However, if we accept a notion of local practice, we can identify smaller-scale 'becomings' in which many more learners do participate. As we saw in examples 1 and 2 above, many learners can become originators of mathematical questions by participating in the practice of asking questions; they can become masters in the use of certain tools (the calculator); they can become masters in operating within a particular set of social constraints. By constraining the foci for attention, and by recognising and working *with* predispositions, rather than ignoring them, a

teacher is more likely to be able to initiate local practices which enable learners to see themselves as members of a mathematical community.

To develop the idea of telos mathematically, in a local practice the telos of individual students could be, for a short while, similarly aligned⁴, just as individual functions may share local approximations. We believe that this alignment of factors is made more likely if the lesson is planned to encourage the development of a local community of mathematical practice.

Conclusion

Looking back to our description on page 2, we can now summarise the features that we believe to be necessary in a classroom if those within it are to constitute a local community of practice:

1. pupils see themselves as functioning mathematically and, for these pupils, it makes sense for them to see their 'being mathematical' as an essential part of who they are within the lesson;
2. through the activities and roles assumed there is public recognition of developing competence within the lesson;
3. learners see themselves as working purposefully together towards the achievement of a common understanding;
4. there are shared ways of behaving, language, habits, values, and tool-use;
5. the lesson is essentially constituted by the active participation of the students and teacher;
6. learners and teachers could, for a while, see themselves as engaged in the same activity.

Our discussion of local communities of practice has practical objectives which include, importantly, providing a language and perspective for beginning and practising teachers as they work to improve the mathematical experiences of their students. However, we think that there are broader issues here which the notion of LCP helps us to raise.

⁴The alignment we have in mind resembles that to which the Cognition and Technology Group at Vanderbilt University (CGTV) refer (1996). They suggest (1996) that in order for children's competencies to reveal themselves a number of elements have to be properly aligned. For CGTV the computer can be seen as an element of a physical and social context which affords or enables 'early competencies' in young children's number. This provides a link between our notion of LCP and the situated abstraction of Noss and Hoyles (1996). Just as they claim the computer provides domains which support students' abstraction, so we claim LCP's support students' growing image of themselves as someone who is legitimately engaged in mathematical practice, as someone, in other words, who is becoming a mathematician.

It would clearly be absurd to claim that only mathematics lessons which have seen the deliberate initiation of local communities of practice can be productive in terms of mathematical learning; it would also be absurd to claim that a classroom is only occasionally a community of practice. We suspect that most successful learners actually experience few mathematics lessons which exploit the ways learning can take place in a community of practice. From our perspective we take this to mean that the success of individual learners will be associated with their positioning within communities of practice, both in and out of the classroom, not as yet described. The initiation of LCPs for which we argue in this chapter, represents a small practical step suggested by our theoretical perspective; the next, much larger step might be to map the complex processes by which some students (far too few) come to value and experience participation in those practices most valued by schools and society, and many do not.

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Chapter 8

SCHOOL MATHEMATICS LEARNING: PARTICIPATION THROUGH APPROPRIATION OF MATHEMATICAL ARTEFACTS¹

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This chapter is based on a research study carried out to understand how students' mathematical knowledge is structured and developed in their mathematical practice in the classroom. It begins by briefly framing this study theoretically and explaining the methodological process of data collection and analysis. In the second part, an episode observed during the data collection is analytically described. Some results of the study on which this chapter is based, especially the results referring to the relationship between school mathematics learning and the process of appropriation of mathematical artefacts, are illustrated.

The research study

We carried out a research study in Portugal in order to understand how students' mathematical knowledge is structured and develops in their school mathematical practice. We were particularly interested in understanding the relationship between students' mathematical learning and the use of school mathematical artefacts in the classroom.

Theoretical framework

The theoretical roots of this study were basically found in three authors and perspectives:

- (i) Vygotsky and activity theory;
- (ii) Lave and a situated perspective of cognition and learning;
- (iii) Schoenfeld and an approach to mathematics learning as the search for a mathematical meaning.

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Always kept in mind and acknowledged was the cultural nature of human activities (including mathematics) and of human cognition.

Within the sociohistorical approach, Vygotsky (discussing the *zone of proximal development*) referred to the interactive nature of changes occurring during a child's development 'in terms of changes in control and responsibility' (Cole, 1985, p.155). This zone, more than a social support, is a *place* where social negotiations about meanings can happen in order for people to appropriate one another's understandings. Vygotsky, in 1934, was trying to understand how cognitive development emerges from institutionally situated activities. He analysed, for instance, how school discourse constitutes a basis for conceptual development. Within the social environment where people learn, Vygotsky included people as well as tools and signs (psychological tools) that mediate social interactions. However, cognitive change does not happen in a closed and determined system but in *systems of social activity* which leads to the decision, within a sociohistorical approach to cognition, to talk about 'individual-acting-with-mediational-means' instead of 'individuals' (Wertsch, 1991, p. 12).

We can find a similar perspective in Lave's 'project' of looking at a *social anthropology of cognition*. In this approach, Lave considered that

cognition observed in everyday practice is distributed – stretched over; not divided among – mind, body, activity and culturally organised settings (1988, p.1).

After observing and analysing how various activities are formed in different situations, Lave does not accept knowledge acquisition as 'context-free' and tries to create a (empirical and theoretical) 'characterisation of situationally specific cognitive practice' (ibid., p. 3). She does not conceive situation and activity as separated from one another and prefers to talk about 'a concept of dialectically constituted, situated activity' (ibid, p. 175). She argues that

a more appropriate unit of analysis will be the whole-person in action, acting with the settings of that activity (ibid., p. 17)

as she conceives that 'setting and activity connect with mind through their constitutive relations with the person acting' (ibid., p. 181). In 1991, Lave states that in order to understand learning it is important

shifting the analytic focus from the individual as learner to learning as participation in the social world, and from the concept of cognitive process to the

more-encompassing view of social practice. (1991, p.43)

To summarise, Lave considers that learning cannot be seen as a kind of activity but rather as an aspect of all activities,

(...) learning is situated in practice as an integral part of generative practice in the lived-in world. (ibid., p.35).

Connected with these ideas we find, for instance, Schoenfeld's perspective. Schoenfeld sees mathematics as an act of sense making which is socially transmitted and constructed. So, we can understand *learning to think mathematically* as developing a mathematical point of view and competence to work with the proper tools in order to understand and appropriate a mathematical sense making. A mathematical sense is no more than a point of view of a given cultural group – mathematicians – which teachers want pupils to learn about and share. Looking at mathematics as an human activity makes us aware of the importance of a 'tacit approval of a more wide community' (Davis & Hersh, 1981, p.60). In saying that, we are not accepting knowledge construction by individual human beings but by human beings that are members of a 'community of belief' (Davis, 1988, p. 12), where human characteristics, such as motives, values and beliefs, play an important role.

Pupils at school learn and use mathematics in their daily activities within a culture which is embedded in their shared meanings and practices. According to Lave (1992),

Math practice in the classroom could be seen as a special cultural activity of its own (p. 87)

and mathematics meaning not determined by the fact that it is *mathematics*, but by its place in schooling – a sociocultural system of activity. In order to understand better the nature of pupils' sense making of mathematical ideas, it seems useful to analyse and interpret their *daily activities* – what people do in daily, weekly, monthly, ordinary cycles of activity (1988, p. 15) – and the *context* where these activities take place.

Some methodological elements

With this theoretical framework, the school mathematical practice of a group of three students in one 8th grade class was observed and analysed during their mathematics classes. The unit of analysis adopted, as proposed by Lave (1988),

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was 'the activity of the persons-acting in setting' (p.177). Videotapes of class observations and interviews with students and the teacher provided the data.

During one month the first author was present in all mathematics lessons. The class was formed by 28 students of a Lisbon secondary school. The teacher allowed the 'invasion' of his classes for a certain period of time by the first author, who is a teacher taking the role of participant observer. No curricular changes were made, neither were there any suggestions of different activities from those which were common in the daily life of this small community - teacher and students.

Lessons usually involved group work or pair work, and it was decided to videotape the activity carried out by a group of two (sometimes three) boys. Earlier on, in a pilot stage, the researcher was present in the class in order to have some idea of the class dynamics and to decide which students would be observed. Criteria for selecting these students were mostly pragmatic. The chosen students easily and naturally accepted interacting with the researcher in a rather similar manner to the way they interacted with the teacher; usually they spoke a lot about what they did; they were considered to be average students in terms of mathematics achievement; their position enabled the video camera to be fixed in such a way that disturbance of the class would be minimal.

We also gathered copies of the work students did in their exercise books. The teacher's speech during most of the classes was taped with a lapel microphone. In addition, we interviewed students, registering these meetings on video (two weeks after ending the observations) and tape-recorded an interview with the teacher.

Data analysis followed an inductive course, which is typical of research centred on classroom phenomena, using data as examples for theoretical discussion. A first form of analysis went on as the videotapes were transcribed; while working these transcripts a first interpretation of the students' activities was attempted. At this point we chose to perform a deeper investigation with tools of conceptual analysis which were inspired by the elements arising from the first analysis. On the one hand, these were consistent with the original theoretical framework and, on the other hand, they were sufficiently manageable, appropriate and useful at this stage of analysis.

Having this in mind, we found that the analytical framework used and described

by Saxe (1991) (see also Colwell and Magajna in this volume) – whose aim is to understand relationships between individual goals² and social life – met our needs and allowed us to find connections between previously examined concepts, helping us give meaning to the first interpretations.

The analysis of practice-linked goals requires, in Saxe's (1991) words, 'an *in situ* analysis of the goals that emerge in practice participation' (p. 23) so 'an ethnography of the practice' (p. 23) is needed, that is we need 'systematic observations of individuals as they participate in their practice'¹. In this study we centred our analysis on three parameters of the first component of Saxe's analytical framework:

- the goal structure;
- social interactions;
- cultural artefacts,

with the aim of reaching some conclusions about the mathematical goals emerging from the observed students' school mathematical practice.

We will now look closely to an episode of the school mathematical practice of the small group of students, trying to give an example of the kind of reflection we have done.

We will focus ourselves particularly on students dealing with mathematical artefacts (in this case Pythagoras theorem) but relating it both with the structure of the practice and with the social interactions (lived within the group and with the teacher).

About artefacts, structuring resources and appropriation

Within the global theoretical framework, we are using the concept of *artefact* as defined by Saxe:

historical products that can be conceptual (for example, the scientific concepts), symbolic forms (for example, numerical system) or material (for example, tools) (1991, p. 4).

So, it seems to us that not only the rulers, compass, calculators, and so on, could be thought as artefacts but also the mathematical objects. In this sense, concepts

²For Saxe goals are: '[...] emerging phenomena, shifting and taking new form as individuals use their knowledge and skills alone and in interaction with others to organise their immediate contexts' (1991, p.17).

(e.g. Pythagoras theorem, mediatrix), methods (e.g. scale drawing), material resources (e.g. compass, ruler) that students need to learn or to use in mathematics classes are taken by us to be school mathematical artefacts. Their history is framed both from the mathematics world and from the school context. Following Vygotsky's (and Saxe's) perspectives conceptual tools play a central and mediation role in learning and with this understanding we were interested, particularly, in understanding how the use of school mathematical artefacts relates to school mathematics learning.

After analysing students' use of conceptual mathematical artefacts (such as: mediatrix, scale drawing, Pythagoras theorem) during their mathematical problem solving activities, we found a kind of pattern we called *students' appropriation process of mathematical artefacts*³. During this analysis both the concepts of 'structuring resource'⁴ and 'mediation' played an important role. We used the term *appropriation* to show the difference between:

- (i) using the artefact as a structuring resource – something that is there to be used (the teacher, the book or copying colleagues' behaviour) and behaving as if it is a proper way of acting in that social context;
- (ii) using the artefact with a mathematical point of view, something that mediates their mathematical approach to new problems.

The former happens when, for instance, students used the artefact of Pythagoras theorem as a result of being pushed by the teacher, their own method not being rigorous enough. The latter was the case when, for instance, students used the artefact of a scale drawing as a good strategy to solve a particular problem (without any suggestion from teacher or the task text).

An episode: the Pythagoras theorem

This episode takes place in a lesson in which students were working in groups solving a problem proposed by the teacher. We will go into some detail

³ Santos, M. and Matos, J. F. (1996) described and discussed one of those processes – the students' appropriation of mediatrix notion.

⁴ To analyse the articulation between different activities and to understand the process that makes possible that the 'same' activity in different occasions could have different meanings, Lave (1988) proposed the concept of *structuring resource* – something (activity, person, objects, etc.) that helps the structuring of a process. So, we can see this idea of structuring resource as something that help us seeing how activity and context interrelate.

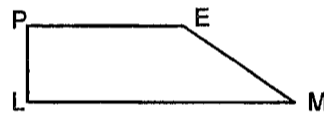
analysing the mathematics activity of a group of students – Tiago, Filipe and Mario. In order to understand this episode other scenes of that lesson, enacted by other students and by the teacher, will be useful too. This is the first lesson in which the three boys were working in the same group.

Classroom situation

As usual, the theme for the lesson was indicated by the teacher at the beginning - *Group work*. No reference was made (in the lesson summary or in the conversation that struck up initially between the students and the teacher) to the mathematical content of that lesson. However, this lesson was integrated into a sequence of lessons dealing with geometrical loci (already identified in previous lesson summaries). A text with a problem was handed over by the teacher to each student on a sheet of paper and the students were encouraged to carefully read the text before starting to work.

Problem

AROUND A LAWN



Mr. António has a lawn in the shape of a rectangular trapezium, in which the bases are 16 and 24 metres long and the height (PL) is 10 metres. At P there is a Pole, at E a Stump, at L an Orange tree and at M an Apple tree (see figure).

- 1) Bobby buried a bone 2 metres from the edge of the lawn and at the same distance from the stump and the apple tree. Where is Bobby's bone?
- 2) To water the lawn, Mr. Antonio has two 'water taps' which send water across the lawn up to 11 metres from the tap. One is next to the pole and the other by the stump. Which part of the lawn does not get watered?
- 3) How far must the taps throw the water to irrigate the whole lawn?
- 4) Mr. Antonio's goat is tied to the stump with a rope. How long must the rope be for it to be able to eat all the grass on the lawn?
- 5) If you put a stump in the middle of the lawn, the goat's rope doesn't need to be so long. In which spot of the lawn can we use the shortest rope?
- 6) In this case, how long is the rope?

In the first part of the class (during which there were, as usual, several behaviour warnings) the teacher made a few short remarks to the whole class about the problem, answering questions posed by some of the students.

Scene 1

A student asks the teacher a question out loud. The teacher then begins a small analysis of the diagram that accompanies the problem with questions that other students answer as well.

- (1) *Student*: The base, is it this base?
- (2) *Teacher*: (*speaking aloud to the class*) For those who can't remember, the
- (3) bases of a trapezium are the 2 parallel sides. So, how long is this one?
- (4) *Student*: 24.
- (5) *Teacher*: 24. How long is this one?
- (6) *Student*: 16.
- (7) *Teacher*: 16, and this one?
- (8) *Student*: 10.
- (9) *Teacher*: 10, and how about this one? [...] We don't know!
- (10) *Student*: Sir, we can make a right-angled triangle.
- (11) *Teacher*: For now... read the 1st question, then do what you need to do.

The teacher's intervention guides the students towards aspects of the diagram that lead them to think about the lengths of the sides of the trapezium. In response, one of the students is aware of the unknown size of one of the sides (line 10) and identifies here a possible problem to solve (or part of the problem) – calculating the measure that is not indicated. It is almost as if he thought that in mathematics exercises one of two things must happen, either (i) all measures of a diagram are given, or (ii) they want you to find out the measure which is missing. However, the teacher chooses to guide students not to analyse that aspect of the situation, but to a methodological aspect – the need to read the text first and the questions, and only then to think about what they need to do. The 'message' he gives is that the question in the text defines 'what you need to do', that is, defines what the students must answer. The teacher's suggestions reveal one of his concerns – to guide the students towards what he believes to be the correct discipline to work in school mathematics. So we are confronted with a teacher attitude related to didactic aspects. However, we must not ignore the fact that this is still the first part of the class (mostly framed by institutional concerns) and the situation is one of dialogue between the teacher and the whole class (where the teacher focuses his concerns on the class as a whole more than on the individual student). The student's reaction (line 10) shows he has spontaneously identified a right-angled triangle as a strategy to find the unknown measurement of [EM]. Thus, he shows that he can identify elements in this situation that justify applying the Pythagoras theorem. In other words, this student reveals knowledge of several aspects of this mathematical artefact.

After this conversation the class continues with the teacher's attention turned towards helping small groups and dealing with direct questions from each of them. We begin to notice the teacher's constant attitude in guiding the students' work towards a geometrical strategy. He encourages drawing the diagram to scale. The text data and conversation (both amongst students and between students and the teacher) focuses on the choice of an appropriate scale for the drawing. It is acceptable to choose different scales, so some groups used scales with squares (they are using ordinary squared paper) as units, others used centimetres (or half centimetres). The teacher keeps reminding the students about how the choice of scale affects the size of the drawing.

Scene 2

The following conversation about scale units goes on between the teacher and a female student in a group:

- (12) *Teacher*: But it'll be very big,... probably it'll be very big. Then we'll
(13) have 24 cm, that could be. If you think it's too big, what can you do?
(14) *Student*: Squares, reduce it...
(15) *Teacher*: Then you end up with... well? You end up with 24 squares
(16) That's more,... more handy to work with. Then you've to
(17) draw circumferences... you'll have very large circumferences.
(18) (*answering a question from another group*) No, each one uses the scale
(19) that best suits him.

We must note that the teacher refers to the need to draw circumferences (and this is not explicit in the problem) as one of the elements to have in mind in terms of the appropriate scale choice. It did not surprise the students that the teacher drew attention to this, rather it was accepted as a good indicator of the advisable size of the drawing. In fact, students know they are in a context of mathematical work related to geometrical loci, which for them are mediatrices and circumferences. So it is a 'shared knowledge' in this class, as often happens in mathematics classes. Mathematical topics follow each other and during the time dedicated to one of them, their work is more obviously related to that topic and less obviously to others.

A previous attempt to reflect on an aspect of the problem that could induce a solution, this time through calculation, emerged on scene 1 as an answer (line 10) to a question from the teacher (line 9).

The system of school activity imposes itself onto the students. This system is marked, for example, by the image of the teacher as the one who has authority to define what the work is about and by the fact that he hands out a sheet of paper

that includes the explicit questions the students are supposed to answer. In scene 2, we can recognise that students accept the reactions of the teacher (line 14) as a sign of recognition of validity in terms that seem to be related to aspects of a (regular) mathematics class within a certain mathematics curricula. The school plays the role of a structuring resource for students' activity providing *guidelines* for that activity that are not induced by the nature of mathematics as a science but by school mathematics as a practice in which they learn to follow a pattern of participation according to the institution. We recognise this kind of school mathematics activity as an institutionally situated activity.

The small group of Tiago, Filipe and Mario

We are now going to describe and analyse what happened in particular with the three students who were systematically observed during the research study, but it is best to give some background information first. Filipe and Tiago are known (by peers and by the teacher) as average or even good students but Mario is not so well rated concerning mathematics learning. This is the first class in which they are all three working together. Tiago and Filipe usually function as a pair even when work is more individual. Usually they discuss what they do and think, being almost a complement for each other – Filipe showing a more pragmatic sense, directed towards quick solutions, and Tiago being more reflexive, questioning the reasons behind choices concerning strategies used. Filipe also demonstrates a greater need to talk to someone in order to think, while Tiago needs moments of silence once in a while, which he achieves by not responding to all of Filipe's requests. This pair has worked like this for some years (since primary school) and they feel it is a good way to work. Today, Tiago is showing a certain difficulty in adapting to work in this new group arrangement. His difficulties seem to stem from confrontation with the fact that there is more dialogue between Filipe and Mario than between himself and Filipe (his work partner of several years). This apparent change of preference (on Filipe's behalf) makes Tiago feel uncomfortable and disturbs his concentration. On the other hand, Filipe has found a good listener in Mario, and this fact, in turn, helps Mario develop a sense of belonging to this group, he has a role to fulfil.

Let us now concentrate on what happens between them during their effort to solve the task presented by the teacher. They decided to do a drawing to scale with no intervention from the teacher and without having turned to other colleagues for help. While they read and interpreted the problem together, the benefit of doing an exact drawing became clear to them. They also discussed the best size for the drawing, emphasising a predictable need of having to sketch

'mediatrices and things like that', as Tiago said. They agreed that the scale would be 1 centimetre to 2 metres and each one performed the task (the drawing and calculations) in his exercise book, constantly comparing with each other and sharing resources and results. They have solved the first and second steps and are trying to solve the third. Everything has been done geometrically, by drawing and measuring, following the teacher's guidance, resorting to and discussing certain numerical aspects only when they need to convert the drawing measures into real distances⁵. Filipe clearly leads the solving process, apparently always with Mario as his working partner. As a result, they reached a solution quickly and with little discussion. Although Tiago follows what Filipe did at almost the same pace, he seems to show some difficulty in understanding the situation involved in the third part of the problem and calls the teacher to clarify the type of taps mentioned in the text. Meanwhile, as the teacher is getting close, Filipe is explaining the movement of the taps to his partner, mimicking with his arms.

Scene 3

The teacher approaches and looks at Filipe's exercise book. The three students look on.

(20) *Teacher*: Which one? 3? They're always there, (*confirming Filipe's explanation*)

(21) *Tiago*: So it's 13 metres.

Tiago turns back to his exercise book, and the conversation goes on now mostly between the teacher and Filipe, but with Mario trying to step in.

(22) *Filipe*: Here it gets to the edge of the lawn, we've to measure from here

(23) to here, it's exactly 13 metres.

(24) *Teacher*: Right (*nods with his head*)

(25) *Filipe*:... to water this area here, so it's 13 metres.

(26) *Teacher*: How do you know it's 13 exactly and not 13.1 or 12.9?

(27) *Filipe (and Mario in a chorus)*: Because we checked.

(28) *Teacher*: How did you check?

(29) *Filipe*: With the ruler...

(30) *Teacher*: Exactly, but since it's a drawing...

(31) *Filipe*: to scale...

(32) *Teacher*:... and not very rigorous, if it were 13.1 it could be the same...

(33) *Filipe*: Mine didn't work out too well, because here I got... 6.4.

⁵ Editor's note: the authors refer to construction and measurement as 'geometry'¹ and calculation using Pythagoras' theorem as 'arithmetic'. I have left these uses as the authors intended rather than adopt the view of geometry as 'properties of shape' currently fashionable in the UK.

Filipe grabs the ruler and starts measuring again as the teacher and Mario look on, while Tiago, who up to that point was redoing his drawing, steps in again.

(34) *Teacher:* 6.4, it's probably 12.8 then! You say it's 12.8, he says it's 13

(pointing to Mario),

(35) *the other one says...*

(36) *Tiago: (stopping work and stretching out in his chair)* I didn't get all that, I

(37) *didn't get it all, I need more...*

The whole of this scene shows how the students are 'stuck' to the resolution involving construction and measuring, even to justify the results they reach. Each of them has found different but reasonably close measures (if we take into account the real situation which is being dealt with). It seems that it is enough that the solution was reached mostly through the drawing (like the real situation described in the problem), in the students' opinion, to justify not minding much about these differences. The teacher's insistence on asking them how they are so sure about the value they reached makes little sense to them. Even the reference to the drawing is used by the students to reassert their confidence, claiming that it was an exact drawing (because it was done 'to scale'). All their answers indicate the thing that is important to them in this process – to see, measure and draw to scale – assuming from the beginning that drawing to scale is a mathematical artefact which legitimates both the process they used and the results they reached. Only after the teacher keeps insisting do they start wondering about the differences in their measurements. Tiago, who up to this point seemed distracted from what was being discussed in the group, seems to perceive an opportunity to regain his position in the group. He makes a point of marking his presence with a certain withdrawal from the sub-group (Filipe and Mario) which was beginning to take shape. He demonstrates this both in his body language and in his insistence on presenting the difference in his result. The teacher takes advantage of the disparity of the results and continues trying to redirect the students' attention to the numerical aspects of the problem.

Scene 4

(38) *Teacher:* Let's see, isn't there a mathematical method that allows us to

(39) *calculate the distance, it's not measuring it's calculating this distance*

(points to exercise book)

(40) *exactly? Didn't we learn a method...*

(41) *Filipe:* Yes we did...

(42) *Teacher:* Which one then?

(43) *Filipe:* Well, we,... always find this and then we...

Filipe keeps pointing to the drawing, which suggests that he is referring to the mediatrix drawing and not to the calculation the teacher has in mind. Mario follows this dialogue (visibly between Filipe and the teacher) and Tiago has already gone back to his drawing.

Clearly they are still thinking about geometrical construction methods, even after having become aware of the teacher's warnings about the differences in their results. In other words, they understand that the teacher wants them to reach a consensual answer, which they have not managed so far. However, they still have not established a connection between the need for exact results and a non-geometrical resolution. It is also curious to observe that Tiago loses interest in the discussion again as soon as it does not meet his aim – to reassert his position in the group.

Scene 5

(44) *Teacher:* No, no, but that's finding out geometrically. But I mean calculating

(45) *Tiago turns his attention again towards the conversation), doing sums and*

(46) *finding out how much this distance measures.*

The teacher points with his fingers to the ends of the straight line segment [EM] in Filipe's exercise book.

(47) *Filipe:* Ah, here it is, the 'cathetus' and the hypotenuse.

(48) *Teacher:* And what do you call that?

(49) *Mario:* The law of...

(50) *Teacher:* You nut-heads!

The teacher laughs, looks at them both, Filipe also laughs and gestures that he wants the teacher to give him more time to remember the name, while Mario practically mimics him.

(51) *Filipe:* Oh, I know his name, Sir, I know his name...

(52) *Teacher:* Theorem of Mr....

(53) *Filipe and Mario:* Pythagoras, there...

(54) *Teacher:* Right, so let's see, we use the Pythagoras theorem to see if this

(55) *really does measure 13. What's this measure? (points to the exercise book)*

(56) *Filipe:* 10 cm, no, 10 metres.

(57) *Teacher:* And this one? You know that too, you can easily calculate it.

A student appears with a question and the teacher turns away from what Filipe and Mario are now doing.

We can see that Tiago is now (line 45) paying more attention to the dialogue between the teacher and Filipe. This shows that he has grasped elements in the talk that are related to another of his aims – understanding the situation. In fact,

the teacher was insisting on making them think about the problem from another point of view, on pointing out connections between mathematical aspects that they were having difficulty in identifying. This was a challenge and a difficult situation which Tiago thought was worth paying attention to. The students only start getting closer to the teacher's idea when he uses words that for them are very tightly related to calculation – 'calculate' (line 44), 'sums' (line 45), 'theorem' (line 52) – or when he makes explicit movements that call their attention to certain elements of the drawing - pointing to the side [EM] – which makes them see a given right-angled triangle (line 47). Then the teacher pushes them toward identifying the name 'And what do you call that?', making the students search in their memories for names linked to right-angled triangles. Finally, they identify Pythagoras theorem and seem to understand what the teacher is after.

Here the intervention of the teacher, in trying to make visible for the students the relation between geometry and arithmetic, has both an institutional and mathematical nature. One of the characteristics of mathematics is the correspondence of results among different fields (in this case classical geometry and arithmetic). From the analysis of the interactions between teacher and students we can conclude that he wanted to underline this feature although this was not explicit in the institutional indicators of this lesson. The interaction between teacher and students in this lesson is not shaped only by the institutional aspects but also by the specific aspect of the mathematics involved (that the teacher wants students to learn within that interaction). We can underline here the fact that this practice is situated both by the institutional aspects and the mathematical content. We have recognised elements that give evidence of the situated nature of school mathematics activity. But the analysis of students' interactions also gives evidence of other aspects of situatedness. For example, learning should be understood in the context of students' participation in the social world of that group (which in turn is part of a broader group in interaction with the teacher). The fact that Filipe (apparently) prefers the interaction with Mario brings a disturbing input on the usual participation of Tiago. A more strong control of the situation is then possible from the part of Filipe. A different kind of development – more speed but less critical evaluation of results, more action and less verbal interaction – imposes to the participation of the other elements of the group ways of doing different from those that seem to be adequate to each one of them. In this kind of interaction Mario finds a position and a role in the group (as a good listener of Filipe) that allows him to participate in the activity in a satisfactory way for his need of acceptance in the group. On the contrary, for Tiago the rhythm seems to be too much, imposing a kind of work less reflective in its global aspects.

Scene 6

Tiago is a little slow, looking in turns first at his exercise book, then at Filipe's work. Filipe grabs the ruler and makes a few measurements as he keeps thinking out loud, without looking or speaking to either of his partners in particular.

- (58) *Filipe*: 8 metres. But we've to do it with this scale. That's it, we'll do
(59) that, good idea. Yeah, you've got to do it according to this (*referring to the measures used in the drawing*).
(60) No, you can do it with the real measures, 10.
(61) *Tiago*: What are you doing?
(62) *Filipe*: Finding this (*points with his pencil to the height from the base [LM]*
(63) *at E*) to make sure this is 13 (*the [EM] side*).

Although it already seems to be clear to Filipe how he is going to use the Pythagoras theorem, he has still not put aside a geometrical approach, as we can tell from what he says (lines 58 and 59). We believe that only when he realises that he does not need to think about the scale does he start 'feeling' the theorem as something which relates measures of the sides of a right-angled triangle and that it might have nothing to do with a previous geometrical resolution. Yet this is still not very clear to this student, as we can tell from his answer to Tiago (line 62). For Filipe, it seems that using the Pythagoras theorem makes sense but only in terms of justifying the results reached, earlier on. That is, he still does not understand how they could have reached the answer to the problem without going through the geometrical sketching. Filipe's perception about using the theorem to check which result of 12.8 m, 12.9 m and 13 m is correct is going to keep jumping up each time he has to explain to Tiago the relationship between using this theorem and the previous resolution.

Scene 7

Tiago had just shown he wasn't '*understanding any of this**' and Filipe starts explaining what he has to do and why, pointing to Tiago's exercise book.

- (64) *Filipe*: To find the correct hypotenuse, which is...
(65) *Tiago*: I know, the Pythagoras theorem... But what does that have to do with this?
(66) *Filipe*: It's to be correct, totally correct The ruler's not enough.
(67) *Tiago*: And if this bit is watered then this bit is too?
(68) (*points to the two corners that were not covered by the 11 metres of the previous step*)
(69) *Filipe*: Where?
(70) *Tiago*: If it waters this then it waters this too?

- (71) *Filipe*: Yeah, we've done it with the a compass. See, it passed here,
(72) ended up right here by the wall, so we already know...
Filipe shows the point where the 2 arcs of the circumference were found, but he does it in Tiago's notebook, who had already drawn them. At the same time Tiago leans back in his chair in a way that demonstrates a certain relief
(73) *Tiago*: So we don't just have to do the Pythagoras theorem, we also have to do this.

Tiago does not seem convinced that the Pythagoras theorem all alone is enough to solve the problem. His insistence results from his need to understand the problem in full, therefore trying to find some sense in intertwining this theorem with the geometrical resolution already carried out (line 65). In his final sentence (line 73) it seems he has already understood the problem as a whole and has attributed a meaning to the steps towards its resolution and to the teacher's suggestions. On the other hand, in his answers to Tiago, Filipe makes it clearer and clearer what purpose he thinks the theorem serves for solving the problem (line 66). He became sure of the distance ([EM]) it was necessary to calculate through a geometrical resolution (lines 71 and 72) but acknowledged that in order to guarantee the accuracy of the measurement of that distance, during that mathematics class, he had to use the Pythagoras theorem. In this manner, he is acquiring the notion that some mathematical methods (as the teacher called them in line 38 of Scene 4), such as the Pythagoras theorem, are tools with which a greater exactness may be obtained than in comparison to more intuitive resolutions, in this case geometrical.

For the realistic situation presented at the problem, there was no need of such accuracy. But that reality was understood by all (students and teacher) as an *excuse* to work on a school mathematical activity, with certain purposes and needs that are different from the ones of the realistic situation. In that context, the teacher led the students into using a mathematical artefact (the Pythagoras theorem) to reach a satisfactory solution to the problem (from the teacher's point of view). For this he used:

- (i) aspects of the resolution already done by the students - the difference in the results;
- (ii) the lack of rigour in the students' geometrical resolutions;
- (iii) key-words that index the arithmetical feature of the resolution the teacher was looking for.

Curiously, the teacher acted the very same way in all of the groups, pointing out the same elements during interaction with students from other groups and using more or less the same words – calculate, sums, method, right-angled triangle – as indicators of what he wanted. However, the meaning given by those three students to the Pythagoras theorem (as a mathematical artefact) in this problem does not seem to relate much to the resolution of the problem itself, but more to the legitimating of the geometrical resolution towards which everything led them, from previous learning experiences (which made them feel the need to draw a diagram to scale) to the structure of the work proposal (in the first and second questions it was asked about places and areas in a way for which a geometrical resolution was adequate). Thus, the Pythagoras theorem (which was actually brought up by a student at the start of the lesson in front of the whole class) was forgotten during the whole geometrical resolution of the problem by this small group. It was brought up once again but that time by the teacher when they already felt to reach the solution for the problem.

Usually, when they arrived at a solution they checked it in the group and also with some elements from other groups (those they trust the most both mathematically and personally). This was what we called the legitimating phase, and during this phase they review and rethink the whole process only if they completely disagree. If there are only small discrepancies they check how it was done by others, what kind of methods or strategies they used and they tend to have a quick understanding of the similarity and acceptability of both resolutions (their own and the colleagues). So in the present case, the use of Pythagoras theorem was associated with legitimating, both by the moment where it was called up and by the way the teacher related it to the students' resolution.

We believe it is possible to identify here the situated character of artefacts, at least in terms of appropriation of their meaning which, for students, is closely related to the relevance and reasons for their use. In this case, we believe they were aware of some of the features of the Pythagoras theorem as a mathematical artefact to justify results reached through a less exact resolution - but the level of use they maintained was still that of a structuring resource. In fact, in this instance its use was visibly forced externally by the teacher's natural 'authority' although it made sense to the students at that moment. The use of the theorem helped to structure the continuation of the activity, but also led to a deeper reasoning about the problem, as we can see from Tiago's statement (line 73). On the other hand, in the midst of this process of appropriation of features of a mathematical artefact like the Pythagoras theorem, the interactions between students in the group were important too because they brought about not only by the mathematical concepts at stake, but also because of each student's

motives, positioning and roles within the group.

Some final thoughts...

This chapter emphasises the relationship between learning in the mathematics lesson and the way students appropriate mathematical cultural artefacts. We tried to identify aspects that reveal the situated character of learning (i) as school learning, and also (ii) as the learning of a specific mathematical content.

The knowledge shared by students (for example, about the possible need to draw circumferences and mediatrices in this class) seems clearly *school-related* as it was shown in scene 2. Similarly, the Pythagoras theorem in this particular problem and at that particular moment, was used by students whose motivation is akin to a school expectation (in which there is a good relationship with the teacher) to give a reply to the teacher's demand. However, this demand is not interpreted by students as a meaningless one. On one hand, the fact that the teacher questions students' results is interpreted by them as a sign that he does not consider their resolution complete in an acceptable way. For the students, the 'authority' in the classroom is clearly the teacher and he is the one who ultimately defines what a good resolution is. But, on the other hand, the meaning they give to the teacher's insistence also concerns aspects that are probably less school-related and more *mathematical*, but even so this reveals a certain attitude towards mathematics.

Students might be satisfied by merely following the orientation of the teacher rather than arguing if the meaning is only school-related. For example they were prepared to argue on the question of rigour which is perceived as something very central in mathematics which they seem to share with the teacher).

In other words, when the teacher points out the differences in the students' results and does not analyse them in the light of the real situation in the problem (lengths of hoses) but rather through the mathematical method used by the students (geometrical method), he is calling their attention to aspects that are fundamentally related to the learning contents of that lesson – mathematics. On one hand, he subtly values reflecting upon the mathematical aspects of the situation that was presented in order to use the mathematical artefact, but he strips it of its real features and, on the other hand, he conveys a different value to the two types of resolution – geometrical and arithmetical.

Apart from these aspects concerning favoured methodologies in approaching problems, we can identify other aspects that point to the *cultural nature* of the teaching/learning process in this class. The teacher keeps encouraging (and the

students appropriating) a certain way of approaching problems, which to him is related to a certain work order typical of mathematics: first read the questions carefully, analyse the information, identify the questions, try to answer them by using their own methods (e.g. drawing to scale), find suitable methods for what they predict may be necessary to use and for the materials they have (drawing to scale, size of the sheet on the notebook and sketching the circumferences). Another aspect revealed in this episode is the idea of mathematical rigour for these students (what allows them to be *sure*) and for the teacher. For students, rigour is associated with the adjustment of a method (for some problems drawing to scale is considered to be an example of rigour), the materials they use (ruler), the sense they use (eyesight), but the teacher emphasises another type of rigour, with a *higher* status, the one which is obtained through calculations.

In this episode, the Pythagoras theorem became closely related to a process of legitimating results. These results may be reached first through other processes, which also have to be rigorous (meaning they have proper rules to abide by) and which are also part of mathematicians' common practice, despite not being looked upon as sufficiently rigorous for this community (the teacher being seen as its representative), making it necessary to turn to other processes of a higher level to certify that results are correct. For the students legitimating is something that reveals a more social need related to school issues – a class where students are working on the same problem; a discipline where it is understood that the solutions for problems cannot be completely different from each other; the very organisation of the class where there was no moment formally devoted by the teacher to the explanation of *the* right answer. We can find here a close relation to the mathematical aspect of the need for a proof or a validation of processes and results. In fact students show a concern for checking processes and results with other groups of students (which was one of the school mathematics goals that were identified).

Cultural artefacts are included in practices, are constitutive parts of (and characterise) those practices, becoming tools (both physical and conceptual) of the practices. Among other ways, people participate in a practice through learning to use and using the artefacts proper to that practice. These artefacts carry with them historical and conceptual elements of the practice. For example, Pythagoras theorem (in its formal appearance) is considered a cultural artefact that belongs simultaneously to two practices: the practice of mathematicians and the practice of school mathematics. In the first one, the theorem carries an historical charge referred to its place in the evolution of mathematics itself and it is associated with a sense of generalisation, symbolic form, something that relates (or shows the correspondence between) two mathematical spaces – geometry

and arithmetic. In case of school mathematics, a parallel can be found to the evolution of school mathematics teaching – Pythagoras theorem can be seen as a *recipe* that the teacher shows when it should be applied or as an important result that can be reached through a process of investigation.

On the other side, the demand for the use of an artefact in a practice can reveal important differences: it emerges from institutional aspects of a practice (for example, the teacher) or from more conceptual aspects – the artefact being *property* of a community as a conceptual tool that mediates the solution (or the approach) to problems of that practice.

To conclude, in order to help students learn mathematics we need to have a better understanding of what school mathematics learning is. In this study, learning basically stemmed from individuals' (students') actions when they participate, along with others (peers and teacher), in a previously structured and institutionalised world (the school) that has its own practice and rhetoric. In this practice, it seems that the central aim presented to students (and felt by them as a community) is their appropriation of school mathematical artefacts. However, during the mathematics lessons students tried to make sense of their own and others' actions and doing so they learned more than just the procedural knowledge associated to those artefacts – they appropriated a certain mathematical knowledge, that is, some cultural aspects of the school mathematical knowledge. When we think of the cultural artefacts of school mathematics practice we are looking at one of the constitutive parts of that practice. Ontologically the concepts of cultural artefact and (social) practice are different. One of the main questions that emerged from this study is the need to go deeper in the analysis of the concept of school mathematics practice (within a situated learning approach and closely connected to Lave's idea of social practice), its main features and artefacts that make it different from other social practices and even from other school practices.

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and arithmetic. In case of school mathematics, a parallel can be found to the evolution of school mathematics teaching – Pythagoras theorem can be seen as a *recipe* that the teacher shows when it should be applied or as an important result that can be reached through a process of investigation.

On the other side, the demand for the use of an artefact in a practice can reveal important differences: it emerges from institutional aspects of a practice (for example, the teacher) or from more conceptual aspects – the artefact being *property* of a community as a conceptual tool that mediates the solution (or the approach) to problems of that practice.

To conclude, in order to help students learn mathematics we need to have a better understanding of what school mathematics learning is. In this study, learning basically stemmed from individuals' (students') actions when they participate, along with others (peers and teacher), in a previously structured and institutionalised world (the school) that has its own practice and rhetoric. In this practice, it seems that the central aim presented to students (and felt by them as a community) is their appropriation of school mathematical artefacts. However, during the mathematics lessons students tried to make sense of their own and others' actions and doing so they learned more than just the procedural knowledge associated to those artefacts – they appropriated a certain mathematical knowledge, that is, some cultural aspects of the school mathematical knowledge. When we think of the cultural artefacts of school mathematics practice we are looking at one of the constitutive parts of that practice. Ontologically the concepts of cultural artefact and (social) practice are different. One of the main questions that emerged from this study is the need to go deeper in the analysis of the concept of school mathematics practice (within a situated learning approach and closely connected to Lave's idea of social practice), its main features and artefacts that make it different from other social practices and even from other school practices.

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Chapter 9

MOVING BETWEEN COMMUNITIES OF PRACTICE: CHILDREN LINKING MATHEMATICAL ACTIVITIES AT HOME AND SCHOOL¹

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Lave (1996a) has recently suggested that we regard learning as 'a long developmental process of moving between multiple communities of practice'. This chapter addresses some issues which arise from taking this perspective, and focuses in particular on the kinds of links or connections which learners might make between different practices. The chapter reports observations from a study in which 32 young children (mean age 6 years 4 months) played one version of a mathematical game with their teacher at school, and another version with a parent at home. Most children spontaneously made connections between the two activities. In some cases children assumed the rules of the game were similar in the two situations, while in other cases children drew on their experiences in one situation to develop appropriate strategies for the other situation. In each location, the adults experienced difficulties in understanding the children's attempts to make connections. These findings are discussed in relation to issues relevant to situativity theory, including the way in which individual learners should be defined and described, the role of learners in making connections between situations, the role of adults (or more experienced participants) in helping learners make such connections, and the nature of the boundaries between different communities of practice.

Introduction

In the last ten years or so, theorists working within the perspective of situated cognition and situated learning have provided a novel and stimulating viewpoint on the nature of knowledge and how it is acquired. These theorists (e.g. Brown, Collins and Duguid, 1989; Chaiklin and Lave, 1993; Lave, 1988; Lave and Wenger, 1991) have explicitly rejected the assumption, underpinning much educational thinking, that knowledge can be separated from the situations in which it is acquired and used. Instead, they have drawn on analyses of everyday social practices, such as grocery shopping in supermarkets (Lave, 1988), to argue that knowledge is essentially situated in these practices, and that it should be seen as an integral part of the specific activity, context and culture in which it is located. In the same way, situated theorists do not see learning as a purely psychological activity, taking place in the minds of learners, but instead view it as a process of increasing participation in particular social practices (e.g. Lave, 1996b).

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One problem for the situated perspective is that of providing an adequate account of what happens when individuals move from one situation to another. Typically, situated theorists (along with others) have explicitly rejected the traditional psychological view that knowledge is some kind of inert substance which is simply 'transferred' between situations (e.g. Lave, 1988). At the same time, there is a recognition in some recent accounts of situated learning that an individual's experiences in one social practice can indeed influence what happens in another practice (e.g. Greeno, 1997). In particular, Lave (1996a) has recently suggested that we 'think of learning as a long developmental process resulting from moving between multiple communities of practice', and that we focus on the 'individual trajectories' of learners as they move between different practices. She also suggests that we pay particular attention to the 'boundaries' between communities of practice, in order to understand more about the ways in which different practices are related and inter-related.

If we take up Lave's suggestion, and think about learning as a process of moving between multiple communities of practice, then a number of questions are raised. For example, we might want to enquire about the kinds of connections - if any - which learners make between different communities of practice. What are the conditions which encourage learners to make such connections, and what are the conditions which inhibit or prevent this process? We might also want to ask about the role which other people (and specifically, more experienced participants) might play in helping learners make these connections. Further, we might want to ask what determines whether two communities of practice are the 'same' or 'different', and how the boundaries between two communities of practice are defined. Are these questions which can be resolved by theorists studying practices from the outside, or do we also need to take account of the perceptions of participants as they move from one practice to another?

In this chapter we present some observations which are intended to illuminate these issues. The observations come from a study in which young children were observed playing two versions of a mathematics activity in two different locations - home and school. Each child played one version of the game with their teacher at school, and another version with a parent at home. The game was based on one used in the widely known IMPACT² scheme (Merttens and Vass, 1993), in which mathematics activities are sent out from school for parents to carry out with their children at home. Our study, it should be noted, was not explicitly designed as a study of 'situated cognition' in either home or school; rather, it was intended as a study of the ways in which parents and teachers interact with children in an apparently similar situation.

The main questions we want to address in this chapter are whether or not the children made connections or links between the two occasions on which they played the game, what these connections were, and what effect these attempts at

² We are grateful to the Director of IMPACT, Ruth Merttens, for permission to use this activity.

linkage had on the adults in each location. Before presenting our data, however, it is worth pausing to consider what predictions might be made concerning these questions from the perspective of situated learning.

One position would be to argue that the two versions of the game - despite their similarities - are in fact quite different activities. One version takes place within the practice of the home, while the other takes place within the practice of the school. Indeed, it might be considered that a fundamental tenet of situativity is that when an activity is introduced into a particular practice, it becomes subsumed - and transformed - by the participants, goals, motives and assumptions which are operating in that practice. Given that the practices of home and school have been known for some time to be very different in these respects (e.g. Tizard and Hughes, 1984), then it might be argued that the two different versions of the game should be regarded as two different activities. Indeed, there is evidence that apparently small differences in the way an activity is 'framed' (Goffman, 1974) can have a significant effect on how that activity is perceived by the participants. For example, Saljo and Wyndhamn (1993) have shown that students will interpret a task involving a postage chart quite differently depending on whether it is presented in a mathematics lesson or in a social studies lesson. From this point of view, then, one might predict that presenting the game within the different contexts of home and school would discourage the children from making connections between the two activities.

An alternative position would give priority to the ways in which the two versions of the game are in fact relatively similar. This perspective would ignore both the different locations in which the two versions are played (home and school) and the different individuals with whom the game is played (parent and teacher). Instead, it would focus on the physical similarities between the two versions of the game (similar materials, instruction sheets etc.), the similarities in how the activity is framed in both locations (similar rules, presence of caring adult, focused attention of that adult etc.), and the underlying mathematical similarity of the two versions. From this point of view, we might argue that the similarities between the two activities outweigh their differences, and hence predict that children would be pre-disposed to make connections between the two situations.

Our current reading of situativity theory is that it is at present insufficiently formulated to enable a clear prediction to be made between these two alternative positions. The study presented here can therefore be seen as providing some empirical observations which can be used to illuminate and extend this particular aspect of the theory.

The study

The study involved 32 children aged between 5 years 2 months and 7 years 4 months, their teachers and their parents. The children came from eight different schools, serving a wide range of catchment areas, and were chosen at random from their classmates. As part of the study, each child engaged in a set of activities with a parent at home and with their teacher at school. The activities consisted of sharing a reading book, playing a mathematics game, and carrying out a science activity involving simple electric circuits. The researcher was present throughout these activities, and after each set she interviewed the child to explore some of the apparent outcomes from the session.

The mathematics game was based on an IMPACT activity, and came in two versions. One version, the Snail Game, shows a diagram of 11 snails ranged around the edge of a patio, (see Figure 1). Each snail has a different number on its back (e.g. from 2 to 12). At the centre of the patio is a lettuce, and there is a pathway from each snail to the lettuce. Each pathway is divided into six spaces, and the halfway point is marked with a darker line. The game is played with two dice which show numerals rather than dots (as we were interested in the representations of number which might be used in the two locations). The players throw the dice, add up the numbers shown, and make a mark on the pathway belonging to the snail with the answer on its back. For example, a throw of '3' and '4' would result in a mark on the path of Snail 7. When one snail reaches halfway (i.e. three spaces have been marked), the children are asked to predict which snail they think will get to the lettuce first. They then continue playing to determine if their prediction is correct.

The other version of the game, the Train Game, is procedurally and mathematically similar but has different surface features (see Figure 2). In this game, 11 trains are trying to reach the buffers which are six spaces ahead at the end of the line. As in the Snail Game, the players throw two dice and add up the numbers to determine which train will 'move' forward: they also have to make a prediction when one of the trains reaches the halfway point. Both versions of the game require simple addition skills, as well as containing some rudimentary notions of probability. For each version of the game there was a set of instructions (see Figures 1 and 3) which explained how to play the game. In some cases the adults read these out to the child, while in other cases the children read the instructions themselves.

In the study, all 32 children played both versions of the game, one at home and the other at school. Counterbalancing ensured that half the children (16) played the Snail Game first, and of this group, half (8) played at home first and half (8) at

Figure 1: Snail Game

Snail trail

YOU WILL NEED: 2 dice and a pen or a pencil.



The snails are trying to reach the lettuce but the gardener has her eye on them !
Which one will get there first?

How to play the game.

- Roll the two dice
- Add up the numbers on the dice and find the snail with that number on it. Make a mark in the space in front of the snail with that number on its shell.
- Continue until one snail reaches the halfway mark then stop and guess which snail you think will get to the lettuce first.
- Which snail gets there first?

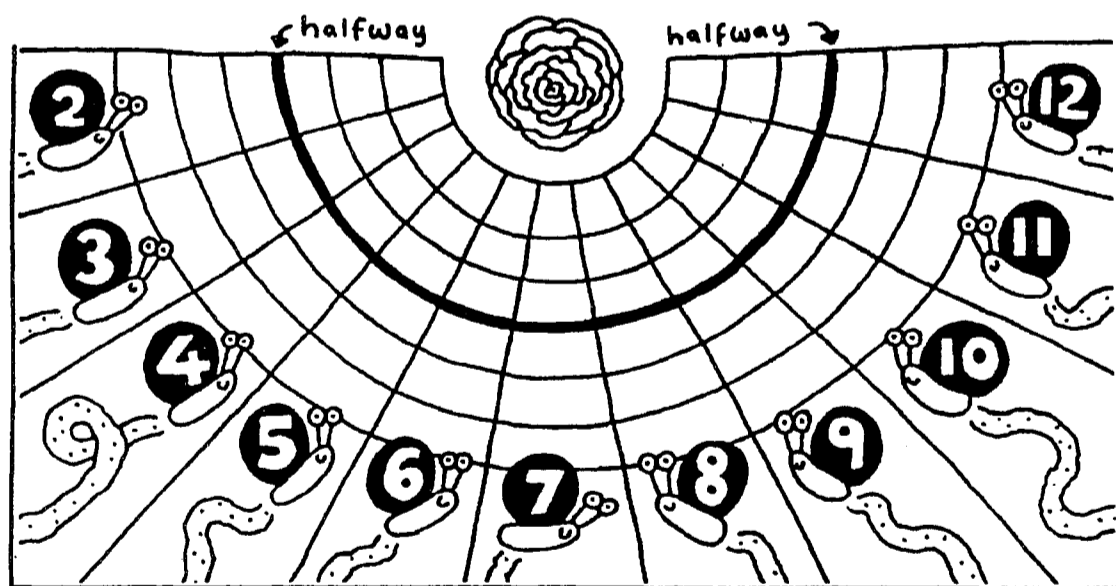


Figure 2: Train Game

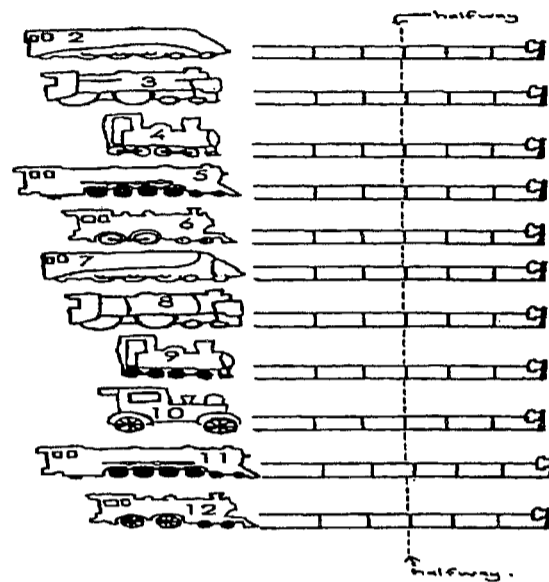


Figure 3: Train Game Instructions

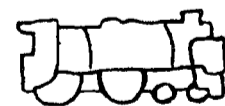
The trains are trying to get to the buffers.

Which one will get there first?

How to play the game.

Each train has a number on it.

- Roll the two dice.
- Add up the numbers on the dice and find the train with that number on it. Make a mark in the space in front of the train with that number on it.
- Continue until one train reaches the halfway mark then stop and guess which train you think will get to the buffers first.
- Which train gets there first?



school first. The remaining children (16) played the Train Game first, with half of this group (8) playing at home first and half (8) at school first.

Children linking situations

All the children in the study, then, came to the second version of the game having first played the other version in a different location. We therefore examined the transcripts of these second playings for any instances of children spontaneously making connections with the earlier playing. This turned out to be surprisingly frequent. Indeed, it was more common for children to make a reference to the previous playing than for them not to.

The children's comments often focused on the similarities in the purposes or procedures of the two versions of the games. For example, in transcript (A) below, the child had previously played the Train Game with her teacher at school. At home, she read the instructions for the Snail Game to herself whilst her mother finished off her coffee. During this reading, the child announced 'Oh I love this'. After reading the instructions a second time, aloud to her mother, she observed:

- (A) Child (G13): Mummy, we did this on the trains.
But we., we tried to do it on., see which train got to the buffer first.
Parent: Oh, and did you have to guess one first then?
Child: We guessed one first and we writ (sic) it down at the top and then when it got to the halfway line, you have to guess which one really won., would win that you thought.

The child's first comment indicated she had noticed that the aim of the game was the same. Her mother's question encouraged her to recount some of the action from the previous playing with respect to how and when predictions were made.

Some children made the assumption right from the start that the games were related. The following conversation took place the moment that the teacher placed the Train Game in front of the child (who had previously played the Snail Game at home)

- (B) Teacher: OK, this one is..
Child: (G7) The train
Teacher: It's the train one.
Child: Halfway.
Teacher: Halfway
Child: What's halfway, which way? That line.
Teacher: Where the line is.
Child: First train to get there.

- Teacher: I think so.
 Child: Because I've had a slug to get to the cabbage before.
 (The previous game had actually involved 'snails' and a 'lettuce', not 'slugs' and a 'cabbage')
 Teacher: Ah, well do we need to read this or not?
 Child: (Shakes head and throws dice)

In this case, the recognition of similarity expressed by the child resulted in the participants deciding to dispense with reading the instructions.

In transcripts (A) and (B), the children made connections that were related to the procedures for playing the game. However, the links made by the children were not limited to procedural similarities. Some children made comments which related to the mathematical ideas underpinning the game. In particular, several children made comments which suggested that, in the course of the second playing of the game, they were reflecting on a decision made during the first playing and were taking account of the outcome of this earlier decision.

This process was particularly evident when one of the snails or trains had reached the halfway stage, and the children were required to make a prediction about which one would finish first. In general, children tended to adopt the strategy of choosing the 'front-runner' (that is, the snail or train which had already reached the halfway point) particularly during the first playing of the game. However, some children for whom this strategy had proved unsuccessful in the first playing appeared to recall this lack of success when they reached the critical point in the second playing. As a result, they rejected their earlier strategy of picking the front-runner, and instead chose a different snail or train. For example, in the following transcript the child had just thrown two numbers adding up to 7, and this brought Snail 7 to the halfway mark ahead of the others.

- (C) Child (G14): Seven
 Teacher: Oo, it's got to the halfway mark
 Child: So it looks like, oo.. I think number eight
 Teacher: You think number eight's going to win?
 Child: Mm
 Teacher: OK
 Child: Cos once, once I did it and like number six was winning,
 Teacher: Mm
 Child: Number six was sort of here
 (points to a backward position)
 then the next time it was up to here
 (points to a forward position)
 and it won.
 Teacher: (laughs uncertainly)

Here the child rejected the front-runner (Snail 7) and deliberately predicted a different winner (Snail 8). She justified her choice - somewhat confusingly - by referring to a previous occasion when a different snail (Snail 6) had 'overtaken' other snails in the second half of the game to finish as the winner.

In the following example, Snail 8 was the front-runner and had just reached half-way. Somewhat to the surprise of the adult, the child explicitly rejected Snail 8 as the likely winner:

- (D) Teacher: Right, so which one do you think is going to win?
 Child (B9): I'm going to choose any one except for eight
 Teacher: Except for eight?
 Child: Yeah
 Teacher: Are you?
 Child: Yeah
 Teacher: Why don't you think eight's going to win then?
 Child: Cause eight, I always choose one like that.
 The first time I played it was like that and I chose the same number, and that one didn't win.
 Teacher: Oh I see. Right so which one are you going to choose then? It's not going to be the eight because it didn't work last time. Which one is it going to be?
 Child: Six

Here, as in transcript (C), the child referred to his prior experience to explain the basis of his thinking. He also generalised from that experience to produce a guiding principle for his current situation. His new strategy was to pass over the current front-runner, because the snail that had occupied that position in the previous playing had then failed to go on and win.

While several children in the study were able to generalise a strategy of 'the front-runner at half-way doesn't always win', there was little evidence of children grasping the underlying mathematical idea that some snails or trains were more likely to win because their numbers could be generated by more combinations of dice (for example, with two dice numbered 1-6, then 7 is the most likely total to be thrown). Perhaps this is not surprising, given the age of the children. However, one child (B9, who appeared in transcript D above) appeared to have achieved at least partial understanding of the mathematics of the game, and was able to generalise this to a slightly different context. In this case, the child was on his own with the researcher after the second playing of the game. They were playing a variant of the game using two tetrahedron dice which displayed the numbers 1-4. This variant (which was new to the child) allowed for combinations from 2 to 8, with 5 being the most likely total. The following conversation ensued:

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- (E) Researcher: Which one do you think will get to the lettuce first?
Child: 4 or 5 or 6. I'll guess 6 again (he had guessed '6' when playing earlier with the teacher - see transcript D)
Researcher: Why did you choose that one?
Child: 'Cos its quite near the middle. It's all quite near the middle of these ones
Researcher: Does that make a difference?
Child: Yeah, that means the more ways you make...that means the more times you're going to shake the number.

Here the child seemed to have grasped that numbers in the middle of the range (i.e. 4,5 and 6) are more likely to be thrown than those which are on the edges (2, 3,7 and 8). However, he seemed unable to go one step further and identify '5' as the most frequent combination.

Adult recognition of linkage

Whilst the recognition of connections seemed to be very natural for the children, its articulation could pose problems for the adults. As they had not been present at the previous playing, they sometimes found it difficult to make sense of what the child was saying. The recognition of linkage was something the adults were unable to share in, and without knowledge of the other situation, they found it difficult to build on the child's comments.

Sometimes, as in transcript (C), the adults exuded an air of puzzled bemusement, ignored the children's comments, and moved on to an item from their own agenda. They also misinterpreted what the children were saying. In transcript (D) for example, the teacher's final comment suggests that she thinks the child is rejecting Snail 8 because he chose the Train 8 on the previous playing, but it didn't win. In fact the child was saying something rather different and more general: namely, that on the previous occasion he had chosen the front-runner (which had not in fact been Train 8), but this had not turned out to be the winner, so he was therefore changing his strategy.

There were also examples of adult and child talking at cross purposes, with the adult failing to hear the real content of what the child was saying.

- (F) Teacher: Which snail do you think will reach there first?
Which snail do you...
Child (Gil): I normally think it's the five, but yesterday five won.
Teacher: Five. So if you put a five there, you think the five will get there first...
Child: He always... I always change my mind
Teacher: So that you can remember which one you think.

The teacher seemed to be wanting a prediction that could be noted down so that they could continue with the game: she thus interpreted the child's mention of 'five' as a choice. In fact the child appeared to be reflecting on her previous experience, albeit somewhat inarticulately, in order to find a basis for making a decision.

Finally, we present a particularly clear example of the adult being unaware of what had happened in the other location, and the consequences this had on the way the child carried out the activity. The example comes from a pilot study carried out before the main study reported here. In this pilot study, the child performed two different versions of an activity in which she was required to generate a total of 25p from a collection of silver coins, which were valued at 5p, 10p and 20p. During the school activity (which took place first), the teacher asked the child what she might use to check her solution. The child suggested she might use a measuring ruler as a number line. The teacher accepted this suggestion, and the child got a ruler which she then used to check her solution. At home, the child asked her mother if she could get a measuring ruler from her bedroom upstairs in order to help her with the activity. Her mother refused, and her comments suggested she saw the ruler as irrelevant to the activity. In other words, the mother's lack of awareness of how the ruler had been used to support the activity at school, together with her inability to imagine it being used as anything other than a measuring device, resulted in her refusing to allow the ruler to be used to support the activity at home.

Discussion

In this study we observed young children playing two versions of a mathematical game in two different locations - with their teacher at school and with a parent at home. We found that most of the children spontaneously made some kind of connection between the two activities. Sometimes these connections were concerned with the procedures for playing the game, with the children making the assumption that the rules for the two versions would be similar. On other occasions the connections were concerned with developing appropriate strategies for playing the game, with several children apparently drawing on their experiences in one location to develop a different strategy in the other location. We also observed that the adults in this study - both parents and teachers - often found it hard to understand the children's attempts to make connections between the two occasions.

In considering the implications of these findings, we need to start with a caveat. As indicated earlier, this research was not originally designed as a study of 'situated learning' in either the home or school contexts. The activity which we observed was not one which took place 'naturally' in either context, but was specifically introduced for the purposes of the study. Moreover, the activity differed from normal and well established practice in both contexts in a number of significant ways. At school, the activity took place outside the classroom

rather than inside it, and the teacher worked on a one-to-one basis with each child for a sustained period of time - an unusual occurrence at school. At home, the practice of parent and child working together on a mathematical activity sent home from school was not a regular occurrence for most of the participants in our study, even though it is becoming increasingly common elsewhere in the UK. Moreover, the presence of the researcher and video-recording equipment added further unusual features to both locations.

In view of these factors, we can draw few conclusions from this study about the normal mathematical practices which take place either at home or at school. In particular, we cannot conclude that young children will spontaneously make similar connections between the mathematics they normally encounter in the classroom and the informal mathematical activities which take place in their homes. Indeed, the evidence from previous research (e.g. Hughes, 1986; Nunes, Schliemann and Carraher, 1993) suggests that young children often have considerable difficulty in making connections between school mathematics and out-of-school mathematics. What the present study does tell us, however, is that the barrier (or 'boundary') between home and school mathematics is not insurmountable, but can be breached under certain conditions. In particular, the study suggests that if the activities taking place at home and school are sufficiently similar - in terms of both their interactional context and mathematical content - then children will make connections between the two different contexts. In other words, the study provides a certain degree of support for the practice of sending home mathematics activities (as popularised by schemes such as IMPACT), provided that similar activities also take place in school.

The study also raises some more general issues which are relevant to situativity theory. First, there is the issue of how individual learners should be defined and described within a theory which focuses primarily on what takes place within social practices. Some of the earlier accounts of situated learning, such as the studies of apprenticeship described by Lave and Wenger (1991), appear to portray the learner in a somewhat passive role, being 'inducted' or 'apprenticed' into specific practices. In contrast, the study reported here portrays the learner in a much more active role, as someone who is explicitly drawing on their experience of previous situations in order to make sense of their current situation. This image of the learner clearly has much in common with the constructivist view that 'learners do not passively receive information but instead actively construct knowledge as they strive to make sense of their worlds' (Cobb, 1996). Such an image is not one which has featured prominently in accounts of situated learning. It may be, however, that Lave's more recent emphasis on the learning as involving the active construction of identities, and her proposal that 'crafting identities in practice becomes the fundamental project subjects engage in' (Lave 1996b) constitutes a recognition of the need for a more active portrayal of the role of learners in situativity theory.

A second, and related issue, concerns the need to provide an account within situativity theory of what happens when individuals move from one situation to another. This need has become particularly acute as a result of Lave's recent suggestion that we focus on the individual trajectories of learners as they move between multiple communities of practice (Lave, 1996a). One of the main contributions of situativity theory has been to challenge the notion that such movement can be adequately described in terms of the 'transfer' of knowledge (Lave, 1988; Brown et al, 1989). However, this critique of the notion of transfer has often been interpreted as the claim that 'transfer cannot take place' (e.g. Anderson et al, 1996; 1997), rather than as a critique of the assumptions underlying the concept of transfer itself. Indeed, several studies from the perspective of situated learning have shown that participation in one community of practice can in fact have a strong influence on subsequent participation in other communities of practice (e.g. Beach, 1995), while Greeno, Smith and Moore (1993) have attempted to develop a theoretical framework for explaining such influences from a situativity perspective. The findings of the present study suggest that such a theoretical framework needs to recognise the active role that learners themselves may play in this process, in which they look for similarities and commonalities across apparently different situations.

This in turn leads on to the third issue, which concerns the role which more experienced participants might play when inducting learners into particular social practices. In some of the accounts of apprenticeship provided by Lave and Wenger (1991), this role appears to be primarily one of performing the role of 'expert' or 'master', together with providing some assistance to the apprentice at appropriate moments. It may be that if we regard learning as participation in a single community of practice, this limited role may be sufficient. However, once we regard learning as a process of moving between practices, as Lave's more recent remarks suggest, then this role may need to be augmented. In particular, there may be a need for more experienced members of a culture to actively help learners make connections across different communities of practice within the culture. However, as our study shows, it is sometimes difficult for an experienced participant (such as a parent or teacher) to play this role if they are rooted in one particular practice, and unaware of what takes place within other practices. This point is perhaps most clearly illustrated by the parent in our study (see p 11) who refused to allow her child to use a ruler as a number line. Here, the use of a particular 'cultural tool' (Brown et al, 1989) to support the child's thinking was encouraged and acceptable in one social practice (school), but was not seen as being relevant or legitimate in another practice (home).

Finally we return to the issue of how communities of practice are defined, and what (or who) determines the boundaries between different practices. At first sight, the issue may not appear to be particularly problematic. It would seem reasonable to assume, for example, that communities of practice are determined by some combination of their physical location, the identities, purposes and goals

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of the participants, the social norms which are operating, and the presence or absence of particular material and cultural resources. From this perspective, then, formal classroom teaching which takes place in school and informal parent-child interaction which takes place at home would constitute two clearly defined and separate communities of practice.

However, Lave's more recent remarks (1996a) indicate a rather different perspective on this. She suggests that there is a field of social relations which encompasses all of our lives, and that we need to question how separateness, boundaries and barriers arise within this inter-connecting field. She continues: 'boundaries do not exist in some natural way - what is it that leads us to feel we are in some bounded practice?'

The observations reported here provide some support for Lave's suggestion that boundaries between communities of practice do not exist in some absolute sense, but are themselves social constructs which are worthy of study in their own right. Thus our example of a parent denying the legitimacy at home of the cognitive tool used in school can be seen as an action which serves primarily to strengthen the boundaries between home and school, and to emphasise the difference between these two communities of practice. Conversely, we have shown that the introduction of a similar mathematical activity in both home and school can serve to weaken boundaries between home and school by encouraging children to make connections between the two situations. Indeed, we would argue that much of what has been studied in the fields of 'parental involvement' and 'home-school relationships' is in fact the study of how the boundaries between home and school are alternatively strengthened and weakened by the often competing and conflicting actions of the main participants in these two communities of practice.

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Chapter 10

THE SITUATED ACTIVITY OF TEACHING AND LEARNING: THE CASE OF INTEGERS

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The authors describe their instructional method 'process-object linking and embedding'. It is designed as a means of overcoming the problem of intuitive gaps recurring in children's mathematical development, and is illustrated here in the case of two attempts to teach integers. It is based on the notion that intuitions arising outside mathematical experience can be imported into the classroom and transformed into mathematics. The design of appropriate classroom activity does need to consider the whole social situation in which the children's intuitions may be stimulated. Authenticity draws on informal knowledge through a simulation in the first and experiential reality in the second experiment. Aspects of situated learning perspectives on the practice of schooling, and on activity in classroom mathematics teaching, are discussed.

Introduction

Recently, work on situated cognition (e.g. Lave, 1988) has developed new social perspectives on teaching and learning in classrooms which adapt concepts such as apprenticeship and peripheral participation. Some of these ideas have directly influenced teaching and the curriculum (e.g. Brown et al, 1989, and The Cognition and Technology Group at Vanderbilt, 1994). Without disputing the relevance of situated cognition to social processes generally, or the relevance of situated learning in Lave and Wenger's (1991) sense to communities of practice, which might include classrooms, in particular, we agree broadly with Heckman and Weissglass' (1994) critique of much of the curriculum work. They say that many examples of curriculum innovation which appeal, perhaps naively, to a 'situated learning' approach have only a little 'authenticity' for children. They see authenticity in the sense one would expect from Lave and Wenger's case studies, i.e. authentic learning is acquired by the individual engaging in a community of practice. A key feature of most daily life and vocational activity is that social goals and purposes tend to subsume learning goals: most learning is picked up by the way, incidentally, from old hands and through experience, (see Billett, 1994 and Wood, 1995).

But the very opposite applies to the classroom, where learning goals are mostly prominent and activity which does not recognise this might be thought inauthentic by students and teachers. The structure of school life as

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an activity system is consequently quite different from vocational and everyday learning situations. To be sure, all learning is situated, and so the process of legitimate peripheral participation is relevant in classrooms: to the induction of children, and new teachers, into school and classroom practices. Such a perspective should reveal a lot about the hidden curriculum, peer group influences, and the incommensurability of school and everyday mathematics (see a number of papers in Chaiklin and Lave, 1993). But this implicit learning is not the overt purpose of education; indeed it is often quite opposed to the learning goals the educational institution espouses. In our view such analyses lead logically to a critical approach to education in school, to a 'critical mathematics education' (Engestrom, 1994), in which the whole social system which contains the classroom is brought into the analysis and into question, and which suggests the school activity must be turned to look to the wider society to motivate its activity.

But in this chapter we argue that it is possible to achieve *some* progress through a partial approach in which children are assumed not to be alienated from the goals of learning mathematics per se, and which attempts to make use of 'authentic' situations from familiar daily life activity, whose sense gives rise to transferable intuitions with which children can build mathematics. Thus we draw on the authenticity of school-learning *and* the authenticity of the children's everyday life and informal knowledge, and hope that each helps to sustain the other. In general the classroom is a community of inquiry, but the inquiry may draw on mathematics from school or outside.

Previously we identified (in Linchevski & Williams, 1996, and under review, and Williams & Linchevski, 1997) some key points in children's construction of mathematical knowledge in school in which they must extend their conceptual structures, such as the operations on the integers and the flexible partitioning of fractional quantities (Semadeni, 1984). Sfard (1991) includes these points in a collection of such gaps which cause problems in mathematical development, where operational conceptions must give rise to structural conceptions, and processes must be mentally reified as objects. She describes the vicious circle which frustrates this development: one must handle the processes as if they were objects first, possibly instrumentally in Skemp's sense, in order to mentally reify them.

We proposed an instructional method which we call 'process-object linking and embedding'. The idea is to link the familiar mathematical processes to objects in a familiar situation, then re-embed the new link through mathematical symbols into their mathematical construction. It makes use of the children's extra-mathematical, ethnomathematical or everyday knowledge to link situations to and 'unpack' processes in the mathematics with which they are already confident. The intuitive development of strategies in the situation then involves representation of these processes literally as 'objects' in

the situation. Through their activity the children are brought to mathematise, and especially encouraged to use the mathematical signs which facilitate transfer into the mathematical 'voice'. Finally the children's activity becomes mathematical, and we speak of the children reifying the processes through their learning to use the new symbols in flexible ways, proceptually (Gray and Tall, 1994).

The problem with teaching negative integers

Negative numbers usually demand an algebraic frame of reference for the first time. While counting numbers are constructed by abstraction from real objects and quantities, and operations performed on them are related to concrete manipulations, operations on negative numbers and the properties of these numbers are usually given meaning through formal mathematical reasoning. Moreover, some of these properties contradict intuitions that have been developed in constructing the counting numbers, (for example, zero is the smallest number!) Over the years this situation has led people in the mathematical community to one of two positions.

One alternative has been to completely avoid any attempt to give practical meaning to the negative numbers, and to recommend treating them formally from the outset (Fischbein, 1987; Freudenthal, 1973). The other alternative is to look for an embodiment, a 'model' that will satisfy the need for providing a practical intuitive meaning to negative numbers, arithmetical operations on them, and the relations between them (we reviewed these studies at some length in Linchevski and Williams, 1996).

The proposed model must preserve the intuitions and schemes that were constructed in the narrower frame and transfer them to the extension. When this condition is satisfied, the person using the model has a feeling of 'correctness'; if it is not satisfied, the person has a feeling of 'fabrication' or 'obscurity'. Inherent in the 'obviousness' criterion is the requirement to avoid artificial conventions that would make a model seem detached from reality. Moreover, in order for the model to fulfil its cognitive function it must describe a reality that is meaningful to the student, in which the extended world (for example, the world which contains negative numbers) already exists and our mathematical activities allow us to discover it. In the specific case of negative numbers this world must include the practical need for two sorts of numbers. It is also necessary to present situations in this world in which the relevant laws can be deduced without 'mental acrobatics' (Janvier, 1985), and without inducing a feeling of contradiction with known truths.

The two experiments we here review were intended to be of this kind, with due attention to the kind of world we thought the children would find intuitive, and with due appreciation of the limitations of replicating an

outside-school situation in a classroom; the relative success and failure of these will allow us to revisit the concept of classroom activity, schooling and transfer of knowledge and comment on social and psychological theories of learning.

Experiment 1: review of the disco-game

In Linchevski and Williams (1996), we described an experiment in teaching the negative integers to sixth-grade students, with an attempt to fulfil the third of Fischbein's (1987) criteria, that of 'obviousness' for addition and subtraction of integers. The construction of the integers essentially involves the construction of an equivalence class of pairs of natural numbers, involving a recognition of the 'sameness' of a class of pairs such as $\{(5,0), (6,1), (7,2)\dots\}$ and the attachment of some label or sign, eventually this will of course be +5. We wanted this to be intuitive. Thus, the integer will attach itself to a situated action (which holds some meaning and can evoke intuition), representations on an abacus (which can be manipulated independently of both the mathematical and the extra-mathematical situation) and some labels, initially just a verbalisation '5 more in', but which in a later episode becomes the formal mathematical symbol, 'plus 5'.

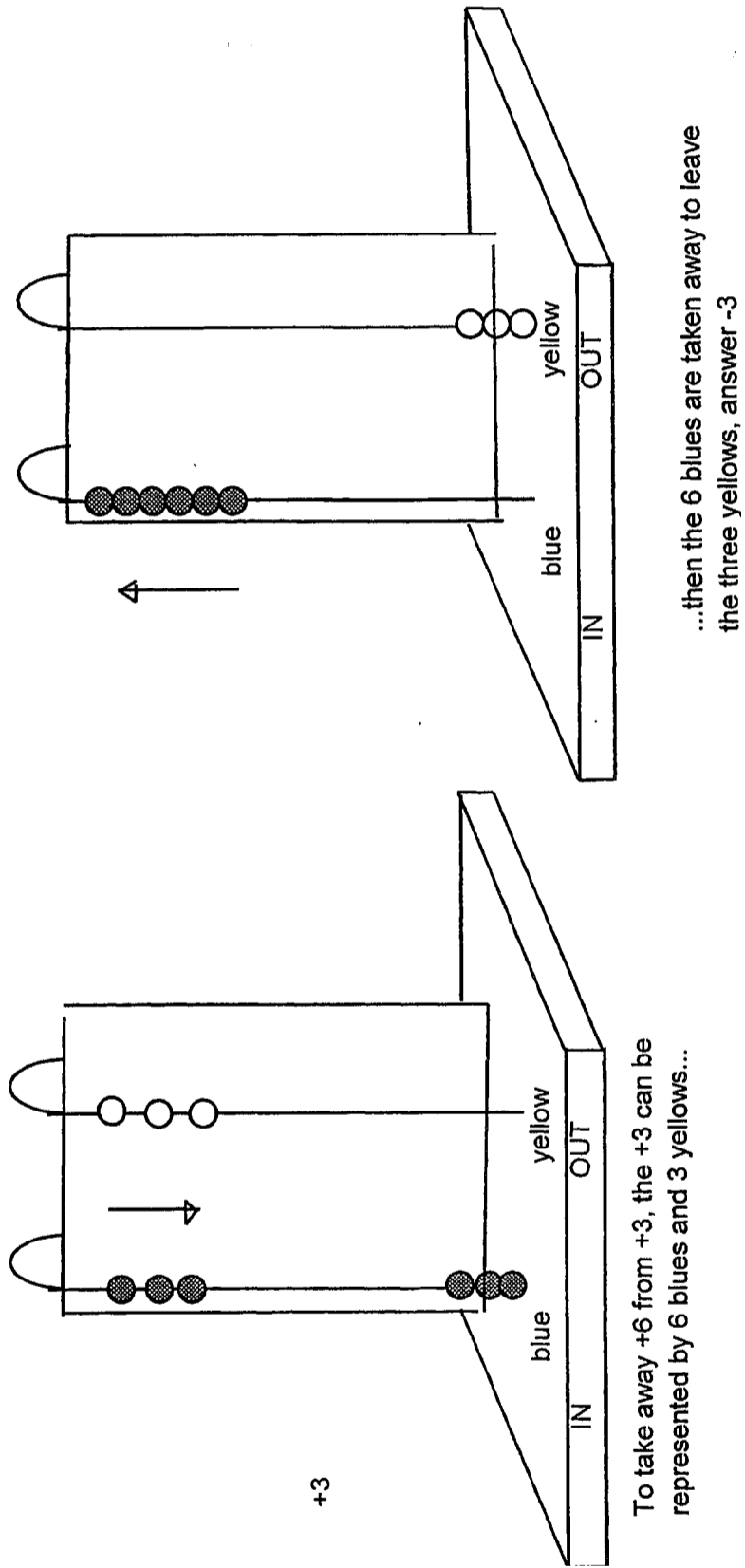
Our teaching followed the approach of Dirks (1984) and others using the double abacus. It was based on a model in which the neutralisation of equal amounts of opposites allows every integer to have many physical representations (Lytle, 1994). We presented the children with a disco-game, a simulation in which the children represent the processes of 'entering and leaving' on an abacus, and so represent these processes as objects (beads on the double abacus) in their activity before they must do so mentally with symbols. The game is played with cards (initially blue and yellow, later these become plus and minus: + and -) which represent dancers coming and going through the disco gates. Each child records the traffic at their own gate using the two bead-colours to count those going in (blue) and those going out (yellow) separately. Each child is periodically required to report the status at their gate, (such as '4 more out', or just '4 out') and combine all the results to see if too many dancers have entered (there is a rule about the maximum number allowed in the disco which ends the game).

Strategies the children developed for dealing with the abacus when it fills up include 'cancellation', where the same number of beads is taken from the 'outs' as the 'ins' (thus maintaining the same status-report) and 'compensation', where a dancer leaving might be recorded by taking one off the blue beads instead of adding one to the yellow beads (for instance if they have used up all the yellow beads). These form the intuitive basis for the operations on the abacus needed later in mathematical calculations. A

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notable result is that cancellation (and its inverse, which we call uncancellation) arose more often and apparently more naturally than compensation, and forms the basis for abacus manipulations which 'go through zero', such as $+3$ take away $+6$ (see Fig. 1). The children will simply add three beads of each colour to the three blues and then take away the six blues to leave the three yellows (-3). The compensation strategy, in which -6 is added, instead of taking away $+6$, rarely arose.

Figure 1: an example of the use of 'un-cancellation' on the abacus to subtract +6 from +3



The children were later asked to check occasionally if the tallying had proceeded correctly by 'taking away the cards' from their abacus. This is the intuitive root of subtraction, and so we can say that subtraction is introduced in the disco-situation as an inverse, but when carried out on the abacus it is a concrete extension of the 'take-away' schema. They take away yellow beads from the yellow pile and blue from the blue, where necessary 'uncancelling', i.e. adding the same number of beads to both wires of the abacus.

They then play with cards which have signs on them +3, -4 etc., instead of colours. They model the recorded value on an equivalent abacus as an integer, represent them in symbols and record the action as a series of sums. The mathematical extension is then more or less complete. But the essential point is that they develop some intuitive sense of the processes and objects as well, and can translate to some extent back to the abacus and situation from the symbols.

Assessment of the success of this teaching (see Linchevski and Williams, 1996) will be clearer in the conclusion of this chapter when we compare the two experiments. In general the children who completed the sequence of instruction were able to perform symbolic calculations of addition and subtraction with few errors. Their calculations in some cases used the abacus and in others not, but all their explanations in response to questioning appealed to the abacus rather than the disco situation.

The degree of obviousness depended on the actual calculation: -8 take away -3 is 'obvious' because you take the three minuses (yellows) away from the eight minuses (yellows) and are left with 5 minuses. But when the calculation 'goes through zero' the explanation is indirect, so +3 take away +8 is less obvious. You have to see that one abacus representing +3 could be 8 pluses (blues) and 5 minuses (yellows), say, so you can then take the 8 blues away and you are left with the 5 yellows, minus 5. This is less obvious: they see it as a calculation to perform rather than an instantaneously obvious result.

When asked to justify calculations 'in the disco' the children were able to do this for additions, but not for subtractions. So adding +3 and -5 is intuitive in the situation. But the inverse involved in the subtraction makes this indirectly formal, the children at this stage did not cope with this. On the other hand the situation has made the integers themselves acceptable as both processes and relations or ordered pairs (-2, the process of 2 going out, and the comparison of 2 fewer after than before), and it has justified for the children the abacus manipulations they will require to 'go through zero', and the equivalence class of abacuses which might allow them to select a convenient representation for an integer.

Experiment 2: review of the dice games

We described in some detail in Williams and Linchevski (1997) the findings of our next teaching experiment with a double abacus but a new situation involving children recording team points scored on the throw of dice, and in which children spontaneously develop the 'compensation strategy' in which points are added to one team rather than subtracted from the other. This is represented and formalised as an intuitive basis for subtracting integers. The abacus facilitates the transfer of the compensation strategy from the situation of point-scoring to the mathematics.

Game 1: A pair of dice (say yellow and blue) is thrown alternately by two teams (the blue and the yellow team, corresponding to the two colours for the abacus beads, and the plus and minus numbers respectively, later on). The scores for the teams are decided by the scores on the dice, and recorded by each team on a double abacus (containing blue and yellow beads). The winning team is the first to get 8 (or more) ahead of the other. The way they record on the abacus is up to the children to discuss: anything goes as long as it is fair to each team. The children may be ready for the next game when they are cancelling the two dice.

Game 2: This time on each turn the yellow and blue dice are thrown as before (and children are expected to cancel the two to a single score for yellows or blues) but there is a third die (labelled add or subtract) which is used to decide whether to add or subtract the result. Thus (3b, 2y, sub) means subtract one blue, and (2b, 5y, add) means add three yellows. Otherwise the scoring takes place as before. The children were encouraged to throw the two coloured dice first and encapsulate the result as say, one blue, or three yellows. Only then, after a moments delay, they throw the die labelled add and subtract. As before we expect to see cancellation and compensation strategies mastered in this new context.

Game 3. We return to the first game and ask how this could be played with only one die. Suggestions may include many interesting games. The one we want to follow up uses a single die with +3,+2,+1, -1,-2 and -3 on the faces. These are interpreted from the point of view of the blues. +3 means 3 for the blues, -3 means 3 for the yellows. The blues win if they get to +8, and the yellows if they get to -8. This game involves consolidating the strategies developed with the blue and yellow dice, but using them with the signed integers.

Game 4. The abacus begins with an equal number of beads on each wire (not empty, usually 9) so that some moves can take place before problems are confronted. Now we return to the second game, but with two dice: the add-subtract die and the signed integer die, (faces: -3, -2, -1, +1, +2, +3). The

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children are, after a time, asked to record games and check them, with the right hand integer column referring to the state of the abacus:

E.g.	Player A	abacus	Player B
	add (+3)	+3,	
		+5,	sub (+2)
	add (-1)	+4,	
		+7,	add (-3)
	add (+1)	+8,	

game ends: Player A wins.

This leads finally to formal sums such as $+3 \text{ sub } (-2) = +5$, and then $+3 - (-2) = +5$.

The main findings of this experiment were:

- team scoring was familiar to the children and they intuitively attended to (a) the difference in the values on the dice as the relevant score for or against their team, (b) the differences in the column heights of the beads as a representation of the difference in team scores.
- they intuitively 'compensated', that is they subtracted from the other team's score rather than added to their own when this was more convenient. This was the crucial intuition on which we hoped to construct the operations on the integers.
- the children found the introduction of the integer signs (to denote scores for the respective teams) to be arbitrary: accepting the minus sign as denoting a score for your own team (e.g. the yellow team) is counter-intuitive.
- the rules of game-playing adopted had certain features which afforded and constrained activity, including the children's communication and mathematical work: they had to justify to each other that their moves were 'fair', but there was also some need to proceed with the game, to 'get on with it'. A significant feature for us as teachers was the fact that games to some extent allow one to capriciously invent, develop or change 'the rules of the game' as and when we require. This allowed us to suggest the extra complications, extra dice and introduction of symbols as and when we felt the children's understanding demanded it.

- in contrast with the 'disco game' the children's understanding of integers that resulted was justifiable in terms of their sense for the game, e.g. Dror explained the sum (+5 - +7 equals -2) as follows: 'I am explaining from the viewpoint of the blues. Plus 5 is like 5 blues and we have to take 7 blues, so we take 5 blues and we add -2, which means 2 yellows/ This depth of understanding, which was quite general, seemed to us remarkable and significant. Indeed we believe that it is just this flexibility, i.e. the ability to perform the symbol rule, act on the abacus and also recall the justification, which may allow for permanent or lasting learning, i.e. relational learning in Skemp's sense.

Comparing the two experiments

Both experiments involve establishing the natural numbers, addition and subtraction in a situation in such a way as to extend their concrete meaning to the integers later. They both involve justifying strategies with the abacus by reference to the situation, and the representation of processes in the situation by objects (beads) on the abacus, which themselves are then manipulated on. But the first led to an intuitive gap at the point where subtraction was introduced (it was a secondary concept, defined by inverse-addition), while the second introduced a gap earlier, when the signs are introduced to refer to teams in an arbitrary way. In this sense both have strengths and weaknesses, and a matching of the two at appropriate stages may be thought sensible.

The situation in the team game allowed the integers to be readily thought of as objects: 'points scored', whereas the integer in the disco situation is most readily seen as a process: dancers going in/out. Thus in the game-situation children seem to refer readily to the integers as objects, which can then be concretely added or subtracted.

Finally an important difference in authenticity appears relevant: a real game is socially valid and carries with it intuitions of fairness which proved important in generating rules and strategies. The disco-simulation was, as such, artificial and introduced some inauthenticity, and did not carry with it such a productive range of intuitions.

It is clear that the second situation carried more experiential reality (in Steffe's, 1996, terms) than the first. The notion of fairness was intuitive because the game was, though simple, a real one for the children. In contrast, the first situation was a simulation, and intuitive ideas about simulated situations do seem to be 'second hand'. This is a fundamental weakness of much of the work of those who call themselves 'situated learning' innovators. The use of even good quality simulation does not carry with it much of the intuitive richness of the social reality. Why is this?

The perception of the children's work as 'activity' within an activity system in Leont'ev's sense (Leont'ev, 1981, Wertsch, 1991, and also in Engestrom, 1996), helps to understand the proposed instructional method. The activity is defined by the goals as well as the tools and language which mediate action. A 'simulation' of a familiar outside-school activity is socially restructured in the classroom as a game with new goals and purposes: its original authenticity is to some extent lost. In the second experiment however the 'game' activity was recalled and reconstituted in the classroom with some of its authentic goals. We believe that intuitions were transferred or imported into the classroom activity because of this.

The integer is constructed in the social activity in a number of ways, but especially it begins as a process on the numbers already understood by the children. Its deeper meaning is formed through the activity, and through the discussion between children, on the social plane, before it is internalised intramentally. Clearly the multiple representations and the use of tools in the activity were very important: this aspect is discussed in Linchevski and Williams (under review). The duality of the integer concept is visible (to us) in the situation presented to the children in the instructional sequence. Then it is visible in the activity of the children, and especially in the language of the children (for instance the process of going in and out is reified in their activity and language when the children speak of the beads representing 'ins' and 'outs'.) Later, we encourage the children to symbolise, to mentally reify the integer, and they begin to manipulate the integers as objects which are added and subtracted. At this point we see the concept has become a mental entity for the individual: reification is complete.

We have generalised the instructional strategy in the three step procedure for teaching certain concepts in which extending the number system might be involved:

1. *Building the link to the situation*
Solving problems posed in the classroom situation should justify intuitively relevant strategies and operations, using only the number concepts and outside school intuitions which are readily understood in making sense of the problem presented. The activity should establish the modelling of the situation and the use of the representations with existing numbers.
2. *Attaching the link to the new numbers*
The new numbers are introduced and symbolised. The intuitive strategies are extended to the new numbers.

3. *Embedding the link*

The formalisation of these strategies and intuitions provide the new mathematical understanding sought: the gap has been 'filled'.

However, the generalisation of our method to teaching using outside-school knowledge to build mathematics might need to be framed more generally. Our conclusions indicate that a teaching method can incorporate the notion of transfer of intuition into school, but that this requires a reconstitution of the knowledge through purposeful classroom activity whose goals are partially structured by the goals of schooling. In particular, the design of a task which the children can relate to appropriately with their prior outside-school knowledge, the use of pedagogical tools such as abacuses which structure recording activity in productive ways, and the introduction of mathematical signs by the teacher are only sustainable in a school institution and a classroom culture in which 'mathematics learning' is the socially supported norm. It is in this sense that we insist that the authenticity of classroom mathematical activity may need to draw on the practice of schooling. This conclusion leads us to comment on aspects of situated learning theory which we find problematic.

Discussion

In this chapter we outlined the development of two teaching experiments in the use of 'modelling' to teach about integers in the didactical phenomenological tradition, in which we attempted to design classroom activity (which is seen as semiotic activity in the children's zone of proximal development) which draws productively on outside-school intuitions as well as their existing mathematical knowledge. In designing the research we were conscious of the problem of 'transfer' of knowledge across situations, and sought to design activities in which children became engaged in reformulating their intuitive knowledge to tackle problems in the classroom which provoked the construction of mathematics. Hence the introduction of tasks which engage children's everyday knowledge, the use of semiotic devices (such as cards, abacuses and written recording) and the introduction of mathematical signs were of key interest in our research.

We have drawn heavily on situated cognition perspectives in this work: intuition is socially situated, activity is structured by the goals, tools and language of the classroom and teacher, and transfer is problematic. However, there are two areas in which we find some situated learning perspectives wanting in our view of the classroom, the first is that of the 'master-apprentice' relationship. We find the master-apprentice model in general distorted beyond the bounds of utility in our analyses. If the key activity-theoretic concepts of activity-motivation and division-of-labour are considered it becomes obvious that modelling the teacher-pupil relationship

on a master-apprentice model causes problems in two key and related respects: first because the 'motivation' of the activity of schools and classrooms is not objectively that of material production, and secondly because the division of labour in a classroom between teacher and pupil is quite sharp. They are socially situated with respect to the classroom, the school and society and have very different goals. The separate goals are, or may be, mutually achieved through mutually constituted classroom activity, but they are not in general 'in agreement' (Matusov, 1996).

Secondly we find that a purely social analysis of learning does not allow us to discuss psychological questions of personal knowledge and intuition. The fact that we may as individuals have personal knowledge to bring to the classroom (e.g. about games) and draw on our personal experience is essential for the process of curriculum development we have outlined. And an analysis of the mathematics in terms of the concepts individuals can construct and use was clearly as essential to the work as our perspective of individuals participating in the activity of discussion and problem-solving together.

We therefore plead, as does Sfard (1998), for learning theories to incorporate the psychological with the social, and for the metaphor of concepts as mental objects to coexist with the metaphors of learning as 'participation' in social processes and in communities of practice. We should not forget that for many children, and for almost all teachers, their participation in classroom activity is contingent on their belief that the child and the class will learn, will gain some knowledge of personal value to them. It seems strange to argue for retaining elements of the psychological paradigm to be maintained given their historic hegemony, but the cause of social theories of learning can only be weakened by an irrational neglect of the relevance of the individual and the psychological.

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Section Four

Endpiece

Chapter 11

LIGHTS AND LIMITS: RECONTEXTUALISING LAVE AND WENGER TO THEORISE KNOWLEDGE OF TEACHING AND OF LEARNING SCHOOL MATHEMATICS

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In this chapter it is argued that Lave and Wenger's social practice theory offers a very powerful language for understanding knowing and learning about and the practice of teaching. However, this theory does not transfer unproblematically into knowing and learning about the practice of school mathematics. This argument arises within a study on teachers' knowledge of their practices in multilingual mathematics classrooms, a study that requires theorising knowledgeability of school mathematics teaching, that is, of both 'teaching' and 'learning school mathematics'. The implications of this argument for research in mathematics education is that Lave and Wenger's social practice theory needs recontextualising if it is to illuminate learning and knowing school mathematics.

Introduction

It is the fate of certain academics to have their work recontextualised out of their field and into the field of mathematics education, and in the process, transformed and all too frequently misappropriated. Jean Piaget is one such to have suffered this fate. And Jean Lave is in danger of becoming another. Among others, Lave's work has been drawn upon by an increasing number of mathematics educators, including myself (see, for example, Jaworski, 1994; Meira, 1995 and other chapters in this book) to look at issues of *the learning of mathematics in school*. This is not Lave's project, even though she has recently written on the issue of learning in school (Lave, 1996), and draws motivation from her analysis of research on learning, that knowledge is not simply internalised and then unproblematically transferred across contexts (one of the key claims about the value of formal school learning, e.g. Scribner and Cole (1973)). However, just as theorising and explanations of learning in school - much of which is unsuccessful - cannot explain successful learning in apprenticeship contexts, so too, a theorising of learning from successful apprenticeship contexts might not be able to unproblematically illuminate or explain success or failure in learning school mathematics.

Lave and Wenger (1991) is a book about apprenticeship learning, seen from an anthropological base. These authors are looking at and describing the

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acquisition of cultural practices taking place in the context of the practices themselves. For example, they look at and describe the means by which novice tailors (want-to-be-tailors) become 'master-tailors'. In consequence, they are looking at processes of cultural continuity where learning is part of the practice, and, in the main, secondary to the tailoring tasks at hand. Lave and Wenger are, generally, not looking at the acquisition of knowledge in formal institutions like schools. These are artificial institutions intent on avowed discontinuity with the practices of the everyday outside their walls. Neither is the focus of Lave and Wenger's attention the acquisition of mathematical knowledge. So why is their work of such apparent interest to mathematics educators?

The ubiquitous problem or challenge for mathematics education is explanation and action that addresses both widespread poor and socially mal-distributed performance. A major debate in the field at the moment is the relationship between formalised mathematics and everyday practices: in particular which sites for learning mathematics are to be offered in school? In current curriculum initiatives in South Africa, for example, one can discern two distinct assumptions about the route to the acquisition of mathematical knowledge: problem-solving activity where problems are 'relevant' and located in everyday or work-place activity on the one hand, and mathematical investigations mirroring the practices of mathematicians on the other. In both instances, we can discern the desire to dissolve the boundary around the school and through this solve problems of access and meaning in mathematical learning. In each case, however, there are practitioners and practices neither of which are to be found in the school. Mathematical practice in school is by necessity neither situated everyday practice nor that of the mathematician. It is a hybrid activity. The challenge for mathematics education is to create a successful cross between these two practices resulting in a strain of school mathematics that is viable in its own right and not pathologising of either practice.

In this chapter, I will examine the theory of learning as social practice as described in Lave and Wenger (1991) to substantiate the claims above and to explore how a non-pathologising recontextualisation of their work might occur. This is not to ignore Lave's more recent and further interrogation of learning as social practice (see Lave, 1993; 1996). Indeed, in the latter paper, she extends her theorising to illuminate learning in formal institutions like school. Rather, it is to signal that in Lave and Wenger (1991) we find a complex set of inter-related concepts that together elaborate a theory of social practice, and so need careful consideration if they are to be harnessed to interpret mathematical learning in school. I will argue that the notion of learning through participation in communities of practice appropriately and powerfully illuminates learning and knowledge about teaching. But a shift into school learning raises questions about

about what constitutes, in Lave and Wenger's terms, a community of practice and its resources, and hence about theorising the learning and knowing of mathematics in school within social practice theory. Social practice theory requires recontextualisation if it is to fully illuminate the complexity of learning and teaching school mathematics. Elsewhere I have argued how sociocultural theory provides such illumination (Adler, 1996).

While the argument in this chapter is theoretical, it arises out of research practice: in particular, a study of secondary school teachers' knowledge of their practices in multilingual mathematics classrooms, aspects of which have been reported elsewhere (Adler, 1995; 1997, 1998). A key challenge in my study became the construction of a conceptual framework that would capture the complexity and tensions in the teaching-learning process and work at two levels: it needed to provide a language to illuminate knowledge of teaching, and at the same time knowledge of school mathematics. Through the chapter I will refer to my specific research as this serves to highlight where a theorising of learning as social practice is illuminating and where it is limited. In this way I hope to provoke an awareness in mathematics education research, that we can obscure that which we are describing if we transport and use parts of a theoretical framework without problematising these in relation to the contexts of their production.

Situating learning in communities of social practice

Lave and Wenger (1991) situate learning in communities of social practice. Building on Lave's earlier work on situated cognition (1985; 1988), they develop a theory of social practice - what they call 'legitimate peripheral participation in communities of practice' (LPP). LPP can illuminate how teachers learn about teaching. It can also be used to throw light on teachers' knowledge about teaching. Lave and Wenger write:

Briefly, a theory of social practice emphasises the relational interdependency of agent and world, activity, meaning, cognition, learning and knowing. It emphasises the inherently socially negotiated character of meaning and the interested, concerned character of the thought and actions of persons-in-activity ... In a theory of practice, cognition and communication in, and with, the social world are situated in the historical development of ongoing activity (pp. 50-51).

For Lave and Wenger, becoming knowledgeable is thus a simultaneous and ongoing fashioning of personal and professional identity within a community of social practice. Learning is seen to be located in the process of co-participation, and not in the heads of individuals. This is thus a social theory of mind where meaning production is located in social arenas that are at once situationally

specific and in the broader society. In Lave and Wenger's terms, knowledge about teaching is thus fundamentally tied to the context of teaching, and cannot be abstracted from it. Knowledge about teaching is also dynamic and simultaneously personal and social.

Legitimate peripheral participation

'Legitimate peripheral participation' (LPP) is the conceptual bridge between the person and the community of practice. As people participate in communities of practice so they become more knowledgeable in the practice. They move from a position of 'newcomers' to becoming 'old-timers' with greater mastery of the practice and with all the conflicts, contradictions, changes and stability that entails. LPP is a means of explaining both the developing identity of persons in the world, and the production and reproduction of the community of practice.

For Lave and Wenger, social practice, and not learning, is the starting point. Learning is rather a dimension of any social practice. It is at once subjective and objective through a focus on whole person-in-the-world. Learning is increasing participation in communities of practices and concerns the whole person acting in the world. This is in sharp contrast to dominant learning theory which is concerned with internalisation of knowledge forms and their transfer to and application in a range of contexts. Knowing is thus an activity by specific people in specific circumstances. Identity, knowing and social membership entail one another. Thus 'learning is not a condition for membership, but is itself an evolving form of membership' (p. 53). Knowing about teaching and becoming a teacher are evolutionary processes, and deeply interwoven in ongoing activity in the practice of teaching. Knowledge about teaching is not acquired in courses about teaching, but in ongoing participation in the teaching community in which such courses might be a part. This view of knowledgeability opens another way of understanding teachers' roles in developing knowledge about teaching. Debates on the 'teacher-as-researcher' often polarise researchers and teacher-researchers, with arguments about what constitutes research, and, moreover, what knowledge about teaching in fact affects practice¹. Lave and Wenger's social practice theory clearly identifies teachers as a crucial source of knowledge about teaching. I am not suggesting here that only teachers can know about teaching. Rather, just as carpenters or tailors are not the only people who know about carpentry or tailoring, they certainly are a key source of any understanding of the practices. Simply, we can and should learn about teaching from teachers themselves.

¹See, for example, Crawford and Adler (1996), Cochran-Smith and Lytle (1993) and Richardson (1994).

Lave and Wenger distinguish between *peripheral* and *full* participation where both are legitimate but different forms of participation in the practice and both are constantly changing. Full participation signals mastery in the form of full membership in the practice rather than an endpoint in learning/knowing all there is to know about the practice. The process of moving from peripheral to full participation thus requires a 'decentering' of mastery and pedagogy away from the individual master or learner and into the structuring of resources in the community of practice (p. 94). Learning and mastery are functions of how resources are made available. For Lave and Wenger understanding participation and learning requires a focus on the learning curriculum, and not the teaching curriculum. It is neither teaching intentions, nor planned pedagogy that can enable and explain learning (p.97). In short, teaching does not equate with learning. Rather, the social structure of the practice and conditions for legitimacy define the practice and possibilities for learning.

Peripheral and full participation provide a model for considering the positions of a teacher in relation to learning. They also provide a means for distinguishing new and older teachers, as well as for distinguishing within newer or older teachers in such a way that those who remain more peripheral are not so simply because they are 'inadequate'. This might well be the case, but must be seen in relation to a teacher's access to resources in the social structure of teaching. The concept of transparency elaborates this point.

Transparency

For Lave and Wenger, learning occurs through centripetal participation in the learning curriculum of the community. Becoming a full member, that is, becoming more knowledgeable, entails having access to a wide range of ongoing activity in the practice - access to old-timers, other members, to information, resources and opportunities for participation. Such access hinges on the concept of *transparency*.

The significance of artefacts in the full complexity of their relations with the practice can be more or less transparent to learners. Transparency in its simplest form may imply that the inner workings of an artefact are available for the learner's inspection...transparency refers to the way in which using artefacts and understanding their significance interact to become one learning process (pp. 102-3).

Becoming a full participant means engaging with all the resources in the community, as well as participating in its social relations. Access to resources - including technologies and artefacts - through their use and understanding of

their significance is crucial. Often material tools are treated as given. Yet, they embody inner workings tied with the history and development of the practice and which are hidden - these need to be made available.

Lave and Wenger elaborate 'transparency' as involving the dual characteristics of *invisibility* and *visibility*²:

... invisibility in the form of unproblematic interpretation and integration (of the artefact) into activity; and visibility in the form of extended access to information. This is not a simple dichotomous distinction, since these two crucial characteristics are in a complex interplay (p. 102).

Access to a practice relates to the dual visibility and invisibility of its resources. In other words, mediating technologies need to be invisible so that they can support the visibility of the subject matter in the practice. For example, in mathematics teaching, the textbook is a resource. It is used widely and often exclusively to teach school mathematics. Its inner workings, however, are undoubtedly tied with the history and development of school mathematics as the acquisition of dominantly procedural knowledge. The text book is highly visible but also invisible in that it makes mathematics (the subject matter) visible. That this dual characteristic of visibility and invisibility can be both enabling and constraining is highlighted in Lampert's study of dilemmas in teaching (1985). A teacher in her study struggled to manage the effective use of a prescribed text book, the inner workings of which revealed mathematics as single methods and answers to problems. Her goals, in contrast, were to enable access to participation in wider conception of mathematical practice. Effective teaching (becoming a full participant) then depends not only on the availability and use of a textbook, but also knowledge of and insight into its history and inner workings, its possibilities and limits.

Extending the concept of transparency further into the classroom and the focus of my study, the example of group discussion of a mathematical task is illuminative. Pupil-pupil discussion of a task should enable the mathematical learning in the task and so be invisible³. However, the rules for constructive

²In the World Book dictionary, adjectival use of transparency refers to the transmission of light so that bodies beyond or behind can be distinctly seen, e.g. a window is transparent. This is the sense in which Lave and Wenger use the term. A window's invisibility is what makes it a window. It is an object through which the outside world becomes visible. That we can see through it, however, is what makes it highly visible. This use of 'transparency' is not to be confused with its figurative meaning as easily seen through or detected, e.g. a transparent lie.

³Meira's analysis of tool use (resources) in mathematics classrooms (1995) distinguishes 'fields of invisibility' which enable smooth entry into a practice, and 'fields of visibility' which extend information by making the world visible.

functioning of a learning group are often left implicit. It is possible that the discussion itself becomes the focus of attention for the group, rather than a means to the mathematics. Here it obscures access to mathematics, by becoming too visible itself. This possibility might well be exaggerated in the workings of a group which has a number of main languages.

In short, practices that are more or less transparent can enable, obstruct or even deny peripheral participation and hence access to the practice. Through transparency, members can exercise control and selection into the practice. Thus, the explanatory burden for learning, and here learning about teaching, is placed in cultural practice. It is placed in the community of teaching, and not on one kind of learning or another. Increasing participation and hence knowledgeability is not about connecting theory and practice, or experience and abstraction, but rather entails the organisation of activities that makes their meaning visible to teachers and all other participants in the practice.

Learning to talk

In addition to transparency, legitimate peripheral participation also involves learning how to talk (and be silent) in the manner of full participants. For newcomers then, the purpose is not to learn from talk as a substitute for legitimate peripheral participation, it is to learn to talk as a key to LPP. Unpacking these concepts related to talk, Lave and Wenger distinguish between *talking within* and *talking about* a practice. Full participation in a community of practice means learning to talk, and this entails talking about and within the practice (p. 109). Talking about the practice from the outside is what often constitutes formal learning (e.g. theory of education in teacher education) where student teachers learn to talk about teaching from outside the practice. For Lave and Wenger this is achieved through a didactic use of language, not itself the discourse of teaching practice, and thus creates a new linguistic practice all of its own.

Talking within and talking about practice thus need redefinition (p. 109). Talking within a practice itself includes both talking within (for example, exchanging information necessary to the progress of ongoing activities) and talking about (for example, stories, community lore). Inside the shared practice, both forms of talk fulfil specific functions: engaging, focusing and shifting attentions, bringing about co-ordination on the one hand; supporting communal forms of memory and reflection as well as signalling memberships on the other. Stories by full participants, say by experienced teachers about their practice, inform other teachers about teaching and demonstrate or model how to tell stories about teaching from within the practice.

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Talking about a practice also usually involves talking both within and about. In Lave and Wenger's terms, the effect of this talk is not full membership of the practice, because it is happening from the outside. It is rather what they call 'sequestration' and an alienation from, or prevention of access to, the practice. We know only too well from teacher education courses that a prospective teacher's ability to write a good essay on what is good teaching - where 'good essay' is signalled in the practices of the academy - often bears little relation to good teaching in practice.

Knowledge about teaching is thus not simply in individual teachers' heads: it is tied to their identities and evolves in and through co-participation in the practices of the teaching community. Teachers, particularly if they have been in practice for some time, are more or less knowledgeable about the practice of teaching⁴, depending on the community, their access to its resources - particularly to activities related to talking within and about the practice, and to the transparency in the practice.

It is this conception of teacher knowledgeability that shaped my study and supported its motivation: teachers have knowledge to share about teaching mathematics in multilingual mathematics classrooms. Lave and Wenger's social practice theory provides a conceptual framework with design implications for a study entailing teachers' knowledge. However, a study of teachers' knowledge of the teaching and learning of mathematics in multilingual school classrooms needs also to theorise knowledge of subject matter (school mathematics) and knowledge of language in use in classrooms. How then does Lave and Wenger's social practice theory transfer from apprenticeships, and other communities of practice like Alcoholics Anonymous and teaching, into school mathematics learning and classroom-based language practices?

Shifting into school mathematics learning

In order to develop an understanding of learning as part of social practice, Lave and Wenger turn to contexts of successful learning - apprenticeships. They explicitly turn away from the school because learning as intended in schools has been unsuccessful for so many and, moreover, in socially distributed ways. In addition, the formal school has been the dominant and determining domain of learning theory, yet it is not the only context of learning.

⁴Of course, all teachers are knowledgeable about their own experience. It is the wider practice of teaching; which is referred to here.

Instead of teachers and learners we have *old-timers* - knowledgeable others in a community of practice - and *new-comers* whose knowledge and identity evolve through centripetal participation in the practice. They elaborate the importance of transparency in the practice and access to resources for newcomers becoming knowledgeable and fashioning a successful identity. I have argued that this conceptualisation of learning within social practice assists the theorising of knowledge about teaching - how teachers learn about teaching. How does Lave and Wenger's conceptualising transfer to theorising learning mathematics (for example) in school? In Lave and Wenger's own terms this question is important: school is a specific social context, involving different communities of practice from those in contexts of apprenticeship.

A shift into school learning raises a number of questions: What/who is the community of practice in school mathematics? What is the community that teachers are old-timers in? mathematicians? mathematics teachers? Or are older students, or mathematically schooled adults the old-timers here? and where are they in relation to the teachers? and pupils? What are pupils new-comers into? Centrally, what might constitute legitimate peripheral participation in the mathematics classroom and towards what is the centripetal process of participation? becoming a mathematician? a mathematically schooled adult?

Lave and Wenger offer a general theory of social practice in which learning is always a part. However, there are clear difficulties as one shifts into the context of schooling. In school, students remain students until they leave. No matter how much mastery they might have achieved, only a few, after school, might become mathematics teachers and even fewer mathematicians. Moreover, their teachers - however mathematical - are not, in the context of schooling, practising mathematicians. Nor are they engaging in ongoing everyday practices. There is also a labour intensity in an apprenticeship model that does not transfer easily to mass schooling conditions. Thus, while Lave and Wenger's intentions are for a general theorising, and they attend at moments (for example, pp. 39-41; p. 100) to the specificity of schooling, they in fact side-step difficulties in using their conceptualisation to interpret and explain teaching and learning in school.

In apprenticeship settings, the main object of the 'master's' attention and intentions, is the practice itself e.g. the making of a suit. A 'good' master will at the same time, enable the participation of the apprentice. In school (perhaps with the exception of art, music and drama), the main object of the teacher's attention and intentions is the learner. A 'good' teacher will be able to bring mathematics in while managing the teaching - learning relationship.

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This difference in objects of attention and intentions suggest that the difficulties in transporting Lave and Wenger's social practice theory into the school can be located in their privileging the structure of the practice in such a way as to exclude the structure of pedagogy (the mediation of knowledge in the relationship between teaching and learning) as the source of learning. For Lave and Wenger, motivation, identity, conflict, power relations, all reside in the community of practice and will work in different ways to enable centripetal movement to full participation or constrain it. This is why learning for them is only understood in relation to a learning rather than a teaching curriculum or intentional instruction (p.40; p.97). But in so doing, and despite their own commitment to move away from dichotomies, they insert a new and equally problematic dichotomy between teaching and learning.

The teaching/learning relation is a hugely complex one. It is as fundamental a problem in teacher education as it is in school learning. Dominant teacher education practices are structured in both the academy and in the school itself - a combination of a formal and an apprenticeship context. The success of this combination and the relative merits, weightings, contents and processes of the two parts remain the focus of ongoing research and debate. Lave and Wenger's theory of social practice shifts the problematic away from theory/practice dichotomies and questions of transfer and encourages us rather to examine the resources made available in different contexts of teacher education and their possible effects. However, in shifting attention onto a learning curriculum and thus correctly questioning any direct relationship between intentional teaching and learning, they nevertheless move to deny any relationship between learning and intentional teaching. This is problematic in general and particularly so in the context of schooling.

Lave and Wenger have, nevertheless, constructed concepts that could provoke interesting insights into learning and teaching mathematics in school. Specifically, *access* and *sequestration*, the *availability of learning resources*, *transparency*, and their distinction between *talking within and about* a practice are easily read into the pedagogical relation in mathematics teaching in school, and are thus useful to explore further.

Language, speech and talk

In relation to transparency and the focus of my study on mathematics learning in multilingual settings, language - and specifically speech - functions as a tool in the classroom. A great deal of classroom communication occurs through speech. Speech is thus a resource where, in Lave and Wenger's terms, invisibility and visibility are in constant interplay: speech should be invisible so that the subject

of inquiry - a mathematical problem, say - can be engaged, i.e. become visible. But language is a cultural tool and never unproblematic. In and of itself, it can mediate the activity in the course of action. For example, a group of learners working on a problem communicate through verbal speech, gestures and so on, about the problem. This communication is supposed to make the problem more visible, more accessible. But the social relations in the discussion and the discussion itself can mediate the problem, particularly if it occurs in a mix of languages. That language itself can mediate activity and obscure the task rather than make it visible seems fairly obvious in a multilingual class. Whether, when and how language used in school classrooms should be transparent, needing to be invisible, yet made visible thus becomes a useful conceptual tool for reflection on and in mathematics teacher education research and practice.

The concept of transparency with its interwoven visibility and invisibility also links, though in a different way, with Edwards and Mercer's (1987) study of classroom talk. They identify implicit rules of educational talk and practice evident in all classrooms. These 'educational ground-rules' are neither arbitrary nor simply imposed by a teacher. They are aspects of culture (p. 59). Successful participation in school is linked to access to these ground rules and they rightfully ask: if these are rules for successful participation why are they implicit? In Lave and Wenger's terms, educational ground rules are cultural resources - they need to be transparent, with the dual characteristics of visibility and invisibility. What Lave and Wenger powerfully illuminate is that resources for learning, like language, can enable or exclude. Depending on how they are used, resources can enable access to the practice or alienate participants.

For Lave and Wenger, becoming knowledgeable in a practice entails learning to talk within and about the practice, and not learning from talk. Yet, curriculum initiatives in mathematics education reflect that it is about both learning to talk and learning from talk⁵. For Pimm (1987) many activities in mathematics classrooms flowing from an interpretation of the Cockcroft report depend on learning from talk, on pupils' verbal expression being seen as an important part of teaching and learning mathematics (p.48).

But Lave and Wenger's distinction between talking within and about becomes useful. First, it links with distinctions between talk as exploratory and talk for displaying knowledge. In mathematics classrooms where there is a move to more exploratory problem-solving mathematical practices, students often work together on tasks, and then report on their working to others in the class and to

⁵This is also reflected in a great deal of literature of language and learning. See, for instance, Barnes (1976) and Britton (1992).

the teachers. While on tasks, pupils could be said to have opportunity for talking *within* their mathematical practice. Then, and either to the teacher, or other pupils or both, they talk *about* their mathematical ideas. Thus they are being provided opportunity to learn to talk but a question that remains unanswered is: given the distinct practice that is school mathematics, that classroom talk has its own form and function (Mercer, 1995), how are pupils apprenticed into this talking? And what happens in classes where children have a range of spoken languages? In short, what Lave and Wenger's theorising of learning does not explain, is the specific demands of apprenticeship into school mathematics, and its necessary focus on the mediation of school mathematics⁶.

Access and alienation

Within a social theory of mind, that is, sharing some basic assumptions with Lave and Wenger, there has been a great deal of research, theorising and debate on the mediation of mathematical knowledge in school. It is beyond the scope of this chapter to elaborate fully here. Briefly, however, more sociological arguments draw on the work of Paul Dowling (see, for example, Coombe and Davis, 1995; Dowling, 1993; 1995) and the importance of the discursive elaboration of mathematical knowledge in the classroom for access or apprenticeship into mathematics as opposed to widespread alienation. Here, mediation of mathematical knowledge via the everyday and the emphasis on procedural knowledge in the curriculum come under scrutiny. More psychologically oriented research has focused on the question of meaning where both children's meanings and socially constructed mathematical knowledge are important in the pedagogical situation. Alienation is a function of the suppressing or ignoring of learner meanings. Informed by both neo-Piagetian and sociocultural theory, quality and effective mathematics learning and teaching in school involve a blending of both self- and other-regulated activity, between scaffolding a mathematical task and providing for creative responses to the task, between intentional teaching and learning activity (see, for example, Cobb, 1994a; Confrey, 1994; 1995a; 1995b). Both cases imply intentional teaching practices where attention is focused on learner meanings and mathematical elaboration.

Explaining access to or sequestration/alienation from school mathematics requires an understanding of pedagogy, that is, of the teaching-learning relationship in school settings. Lave and Wenger's social practice theory falls

⁶In language learning terms, the distinction that is missing in Lave and Wenger's work is that between acquisition and learning. Some language has to be learnt (and thus needs a focus on the structure of pedagogy - on its mediation), some language is acquired (in the structure of practice).

short here. This argument is strengthened by a brief elaboration of the nature of school mathematical knowledge as a discourse.

School mathematics as a discourse

There are two understandings of school mathematics embedded and alluded to in this discussion. The first is that the school is a specific context. The learning and teaching of mathematics in school is thus a quite specific social practice.

The second is that school mathematics needs to be understood as a discursive subject⁷ or as a set of discourses, where 'discourse' means 'language as it is used to carry out the social and intellectual life of a community' (Mercer, 1995, p. 79) where the mathematical register (Pimm, 1987) is part of the discourse. From this perspective, learning mathematics entails acquiring, recognising and developing specific ways of using language, or, in Lave and Wenger's terms, learning to talk. Furthermore, school mathematics is learnt through discourse, through language in use in the classroom. An important question arises in relation to school knowledge. What is the discourse of school mathematics? It is not the discourse of mathematicians - they are in a different community of practice from that of a classroom. School mathematics is also not the discourse of apprenticeships, nor of the everyday. School mathematics is a social practice with specific time-space relations, activities and discursive practices. School mathematics is a distinct practice (Muller and Taylor, 1995; Dowling, 1993; 1995)⁸, a hybrid where there are recontextualisations from the discipline of mathematics and its applications into the curriculum.

Mercer provides a language with which to understand the special nature of classroom education and knowledge produced in the context of schooling. He distinguishes between *educational discourse* - the discourse of teaching and learning in the classroom - and *educated discourse* - new ways of using language, 'ways with words' which will enable pupils to become active members of wider communities of educated discourse (Mercer, 1995, p. 82). In Mercer's terms, educated discourse in school mathematics will include the mathematics register.

⁷See Pimm (1987, p.47) for discussion of how, typically, teachers do not conceive of mathematics as a discursive subject - it is not something that can be discussed, that learners could have opinions about. It is a matter, rather, of right and wrong answers to given problems.

⁸Dowling, and Muller and Taylor, provide a sociological argument about the distinctiveness between school knowledge and everyday or 'relevant' knowledge. For Dowling, school mathematics has high discursive saturation (it is embedded in discursive relations); everyday knowledge, in contrast, has low discursive saturation. Everyday and school mathematical knowledge are thus incommensurate and attempts to teach mathematics through everyday meaning and relevance are likely to exclude learners from mathematical knowledge.

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Learners can develop familiarity and confidence using new educational discourses only by using them. While pupils all engage in educational discourse, they need opportunities to practise being users of educational discourses. Often there is a mismatch between the educational discourse in play (ways with words in the classroom) and the educational discourse they are meant to be entering. The teacher's role is to translate what is being said into academic discourse, to help frame discussion, pose questions, suggest real life connections, probe arguments and ask for evidence. This does not mean that the teacher's role is simply to explain, but more to be the person who brings the language and the frames of reference of the 'expert' discourse into the 'collective consciousness' of the group (Mercer, p. 81). The language practices of the classroom (educational discourse) must 'scaffold students' entry into educational discourse' (p.82). This is not a negation of student creativity. Even for creativity, students still need to know the discourse.

Teachers are expected to help their students develop ways of talking, writing and thinking which will enable them to travel on under intellectual journeys, understand and being understood by other members of wider communities of educational discourse: but they have to start from where learners are, to use what they already know, and help them go back and forth across the bridge from 'everyday' discourse into 'educated discourse' (Mercer, 1995, p. 83).

It is in this understanding of the aims of school education that Lave and Wenger's seamless web of practices entailed in moving from peripheral to full participation in a community of practice is problematic. Their continuity argument denies the crossing of any bridges. What do talking within and talking about then mean in classroom practices? How do learners learn to talk about mathematics? That there is a bridge to cross between everyday and educational discourses is at the heart of Walkerdine's (1988) argument for 'good teaching' entailing chains of signification in the classroom where everyday notions have to be prised out of their discursive practice and situated in a new and different discursive practice.

Current debate in mathematics education, stimulated by more fallibilist conceptions of mathematical knowledge, reflects attempts to change the quality of experiences learners have away from the procedural application of rules to more principled, deliberate thinking, problem-solving and problem-posing. The goal is, on the one hand, to make school mathematical experiences more authentic, more like the mathematics of mathematicians, and on the other, to bring in the real world of problems and applications. Solving a mathematical problem in school is not simply continuous with solving mathematical problems in other real world contexts. Nor is school mathematical practice the practice of

mathematicians. It can and should include both. But it will always be through a recontextualisation of and a successful cross between these different practices. Hence the need, conceptually and practically, for a focus on pedagogy in the mathematics classroom to enable the crossings between discourses, registers, and between languages and social situations. And crossings in the classroom create significant challenges for teachers. As Muller and Taylor (1995) argue, such crossings can be dangerous and alienating in school and more for some learners than others. As mentioned earlier, I have argued elsewhere how sociocultural theory can illuminate the specificity of mathematics teaching and learning in multilingual classrooms.

Conclusion

I have argued that mathematics teaching is complex. Thus being able to describe and explain teachers' knowledge of their practice in such contexts (and so learn from teachers) requires a language of description that embraces this complexity. Lave and Wenger's social practice theory is powerful here.

Becoming knowledgeable (both of mathematics in school and of teaching) is bound up with access to resources in the practice through their transparency - their dual characteristics of visibility and invisibility. Language is a resource in the classroom. Teachers in multilingual mathematics classrooms thus need to work between the languages learners bring to the class and the language of instruction. There are tensions in this. Becoming knowledgeable also entails learning to talk (in and as part of a community of practice) where learning to talk includes both talking within and talking about a practice. However, the shift from talking within to talking about mathematics in school is not a seamless web, but one that requires mediation. In all classrooms, and particularly in multilingual classrooms, it is the teacher's role to enable learners to move back and forth between talking within and about mathematics, between educational and everyday discourses in the classroom, and between everyday and school mathematics. In short, explaining access to or sequestration/alienation from school mathematics requires an understanding of the structure of pedagogy. Lave and Wenger's social practice theory falls short here. A non-pathologising recontextualisation is necessary.

The recontextualisation offered in this chapter is situated in an interpretation of school mathematics as a hybrid activity where teachers' objects of attention and intentions are quite different from those of old-timers - 'masters' - in apprenticeship contexts. The recontextualisation is an integration of the structure of pedagogy, of mediating bounded discourses or bridge-crossings, as

offered by sociocultural theory, with Lave and Wenger's notions of access to a practice through the transparency of its resources and through learning to talk.

Perhaps the problem of learning as addressed in this chapter lies in our endless searching for a monolithic explanation of learning. This would explain Lave's (1996) interesting attempt to elaborate social practice theory into the school. Here we are offered important insight into the learning about and formation of personal, including racial, identities through schooling practices, but still little that addresses the issues raised in this chapter in relation to mathematical learning in school. Perhaps learning is, after all, not a unitary phenomenon, and thus not amenable to one all-embracing theory.

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