**Deep Progress in Mathematics**

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In this article I will introduce some ideas associated with the Improving Attainment in Mathematics Project (Watson, De Geest and Prestage 2003), in which ten teachers made a commitment to raising the expectations of previously underachieving 11-12 year-olds in English schools. This work took place in the English context of frequent testing; a prescribed curriculum; official guidance about best lesson structures; official schemes of work; and the segregation of children into different mathematics groups from as early as the first year of school, in some rare cases, and certainly by year 7 (the first year of secondary school) in nearly all cases. The messages which emerged from their work were, however, universal although I do not pretend to know the Scottish context well enough to suggest how they might apply.

Typically, groups of low attaining students are taught simplified mathematics, often in a step-by-step manner and dressed up in pseudo-relevant contexts (Boaler, Wiliam & Brown 2000; Wiliam & Bartholomew 2004). Often they are expected to work in different ways than other students. For example, they may be expected to discuss less often because they find discussion difficult for a range of social, linguistic and psychological reasons; they may be expected to copy given methods rather than construct their own, because their own are more often based on simplistic assumptions than the methods constructed by those in other classes; they may be expected to practise techniques until they can do them accurately, where students in other groups may ‘get away with’ more errors because their underlying understanding seems to be fine.

The project teachers believed that adapting mathematics and mathematical engagement to fit these limiting aspirations was not only wrong ethically, it was also ensuring low attainment, rather than improving it. They held a set of alternative beliefs that:

* *all* students can think hard about mathematics, and thus do better at mathematics. Indeed, they also realised that low attaining students already think more effortfully than others when they do mathematics, because it is harder for them to understand it.
* all students have the right to, and are capable of, full engagement with the subject. So low attaining students should be offered the full curriculum, not just the technical parts. There are two ways to interpret this: one way would be to agree that low attaining students should, like all students, learn geometry, handling data and algebra; the other is that they should learn maths which is as hard as the maths taught to others. In practice, the project teachers took both the ‘spread’ and ‘depth’ views.
* all students needed to learn maths in ways which develop reasoning and thinking and confidence in problem-solving
* all students are entitled to have access to the maths necessary to function in society, beyond minimal social arithmetic.
* learners can change their goal from ‘to finish’ or ‘to fit in’ to ‘to learn’
* examination and national test results are important, so good results is an aim alongside giving breadth to their mathematical experience
* changing habits is hard for teachers and for learners
* success in maths can be a source of self-esteem and empowerment.

These are the beliefs which many people have when they enter teaching, but the immense difficulty of enacting these beliefs is daunting. It becomes easier to accept habits of low expectations than to be constantly struggling against defeatism. Too often the discourse of staffrooms allows talk of ‘low attainers’, describing students rather than their behaviour, dwelling on their deficiences by saying ‘they can’t concentrate’ ‘they won’t do homework’ ‘they can’t think for themselves’ and so on. To make changes requires energy and courage, and the project teachers found that being part of a group helped this to happen.

My own starting place comes from our natural propensities to look for patterns and even to impose them on our sensory experience so that we can begin to make sense of what we see. This leads immediately to questions for teachers: how can I present concepts using patterns? Can I control variables so the ideas are easy to see?

It is also natural to match our ideas to those of other people through discussion, or through watching and listening to others. Many low attaining students find that their ideas seldom match others’, in particular the teacher’s, and hence opt out of learning. Given the opportunity to express their ideas and have them heard and understood, learners can become re-included in the group learning process. The questions for teachers are: what can I learn from listening to students who do not immediately understand? How can I use the matching of different perceptions in lessons?

Another natural propensity is to use examples to express our thoughts, rather than speak generally or abstractly. In mathematics lessons learners are often expected to do the opposite, to speak abstractly when given examples by others, yet the creation of examples encourages direct engagement with concepts and allows for more revealing communication. How can learners’ own examples be incorporated into lessons? I have written, with John Mason, about these features of the human mind, and how they might be incorporated into good mathematics teaching (Watson & Mason 1998 & 2005).

In the project teachers were not told how to teach, nor given any other kinds of instruction. The only guidance was to teach through developing mathematical thinking and to focus on key mathematical ideas, yet they were free to use their own interpretation of these phrases. They developed their own practices supported by the discussions of the group and the introduction of some general resources from time to time which reflected the nature of their concerns (e.g. Prestage & Perks 2001; Ollerton 2002).

In due course a long list of the kinds of classroom activity which, in the teachers’ view, developed mathematical thinking was created by the teachers. It was clear that this related also to the development of deep understanding of mathematical topics. There was no separation between tasks which encouraged exploration and those which focused on learning traditional content. Exploration was often used to contact standard curriculum content, but there would be other times when practice for fluency, or the development of memory were also thought appropriate. Teachers would distinguish between aspects of mathematics which needed to be fluent; aspects which needed deep conceptual understanding; aspects which might lead to linking and consolidation of previous knowledge and so on. By observing teachers and discussing their planning we found that learners were usually encouraged to:

* Choose appropriate techniques
* Give reasons
* Share their methods
* Pose their own questions
* Deal with unfamiliar problems
* Contribute examples
* Describe connections with prior knowledge
* Identify what can be changed
* Find similarities and differences beyond superficial appearance
* Make something more difficult
* Make comparisons
* Generalise structure from diagrams and examples
* Predict problems
* Work on extended tasks over time

Here is an example of one project teaching sequence. The idea came from Ollerton’s book (2002) (though he does not claim originality). Sara, one of the project teachers, asked her class to cut out some congruent squares, and having done so to fit them together, edge to edge, to see what perimeters they could make.

At the end of the first lesson, learners put their squares into individual plastic wallets, which were re-distributed to their places before they came in for the next lesson. This sort of practice is well-known in primary schools, but not so common in secondary schools. It provided a model of personal organisation and a means of emotional attachment to the work. Further, handling pieces they had made themselves triggered recall of the work done before, so there was less need to ‘settle’ the class and remind them about what they had been doing.

The task allowed for simplifying the problem by working with two, three or four squares to establish rules about what kinds of joining were allowed, to develop subgoals, and to become fluent in these as the number of squares increased. Thus learners who had found it hard at first began to find some aspects easier as they increased numbers of squares, while facing new challenges. A sense of personal progress and increased knowledge was thus embedded into the task. Some were able to recognise their shifts of focus as increased skills.

If edges were allowed to meet in part, rather than as wholes, the task gave the opportunity to work beyond whole numbers and even to appreciate the notion of a limit. Learners were thus able to relate this work to fractions as numbers on a numberline.

Two important aspects of Sara’s teaching were that the task was allowed to run on and on, even when there might have been times when nothing much was being achieved. She felt the experience of working in an extended way was important. Another feature was that, although they had been allowed to access the task ‘at their own level’, she tried to find ways to move them on from there to new realisations and skills. Thus she demonstrated many of the beliefs and prompts listed above.

By analysing the kinds of progress which teachers wanted their students to make, we constructed the notion of ‘deep progress’, which means that learners do any or all of these, in no particular order:

* Learn more mathematics
* Become better at learning mathematics
* Feel better about themselves as mathematical learners

By aiming at all of these outcomes during the project, teachers were able to report that there had been progress in classroom participation, groups became more enjoyable to teach, and learners were more interested in mathematics. They also did better in tests and in exploratory tasks. However, I do not want to paint too rosy a picture: these were real students with real problems of behaviour, self-esteem, often with erratic lives, language difficulties, poor study habits and so on. In one class ‘progress’ was that two-thirds of the students were participating by half-way through the year, when only a quarter had at the start of the year. In another class, students had to be re-trained in a punitive way to take work home and bring it back. It was as if they were unaware that they were even capable of doing that, and had to be shown that they too could act like self-organising learners.

By teachers being determined, brave and consistent, the classes began to show that students could take responsibility for spontaneous mathematical thought, beyond superficial engagement. Indeed, many of the prompts teachers had used to ‘get them to think’ became things learners would do for themselves from time to time. Increased independence led many learners to shift to new work and thinking habits:

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| --- | --- |
| **Teachers would ask learners to:** | **Learners would take the initiative to:** |
| * Make something more difficult
* Make comparisons
* Pose their own questions
* Predict problems
* Give reasons
* Work on extended tasks over time
* Share their methods
* Deal with unfamiliar problems
 | * Make something more difficult
* Make extra comparisons
* Generate their own enquiry
* Predict problems
* Give reasons
* Spend more time on tasks
* Create methods and shortcuts
* Deal with unfamiliar problems
* Initiate a mathematical idea
* Change their mind with new experience
 |

Although the teachers were all very different, there was convergence towards some common habits as the project progressed. All teachers found themselves giving learners more time, either by waiting a long time for answers, or by spending a long time on each topic (two weeks on subtraction in one case), or by (as in Sara’s examples) letting activities go on and on, even when pace seemed to slowing down. The teachers began to expect low attaining students to be as self-directing as their other groups, even when unhelpful moves were sometimes made. All teachers found themselves offering tasks more often in visual, oral, tactile, shared modes rather than from textbooks or worksheets. But this does not mean they were simplifying mathematics, far from it. The idea was to make complex, rigorous mathematics available through accessible media and modes, and to discuss the complexity and rigour explicitly.

Task design is obviously an important component of this approach, and I admit that we did not make as much progress as it is possible to make during the project itself. I can offer an example of a teacher who did achieve extraordinary learning outcomes through clever task design, and refer the reader to other sources to find out more (e.g. Watson & Mason 1998, Mason & Johnston-Wilder 2004; Bills et al. 2004 )

Rebecca was a student-teacher. I saw her teach one Thursday afternoon, a lesson with ‘bottom set’ year 10 in a very typical comprehensive school, the day before the end of term, after a lunchtime which had included a fight. The group were destined to do the lowest tier of GCSE, and hence simultaneous equations were not a major focus for them - and they knew it! However, Rebecca thought she would try to teach this topic, with no attempt to dress the work up into some kind of context.

She had prepared this diagram using powerpoint:

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No one asked ‘why are we doing this?’ and they all understood that their task was to sort out what a triangle was worth. About half an hour into the lesson, the class were still paying close attention and beginning to solve more complicated situations such as:

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In this way, Rebecca had introduced them to ideas of substitution and elimination by using their natural propensity to engage with ideas, to unravel puzzles, to imbue diagrams and symbols with meaning, and to make sense of experience. Thus they engaged with simultaneous equations.

A particular feature of Sara’s and Rebecca’s tasks was the control of variables, so that learners were encouraged, visually, physically and verbally, to focus on the effects of varying within a constrained system.

I have also seen this same kind of high-level challenge used for other groups. A class of 14 year-olds who had taken 16+ examinations two years early were given this equation to solve:



This was way beyond any curriculum expectations, and the students were not an especially gifted class – they were a quarter of the year cohort of an inner-city comprehensive school which had immense social diversity (for instance, 50% of the students had free school meals). The philosophy was to immerse them in complex mathematics and then to work with them as a group to identify how to bring prior knowledge to bear on the task. Many of the features of the project teaching were visible in this class, in particular the pooling of ideas and finding similarities and differences with other experiences. This last example was a closed question treated as an arena for exploration, as was Rebecca’s simultaneous equations. Sara’s task was more open, yet with all three similar kinds of interaction developed in the lessons.

Teachers such as Rebecca and Sara seem able to act in very different ways to what is normally seen in such classrooms, maintaining integrity with mathematics and with adolescent students in ways which should, if sustained, lead to higher achievement and higher self-esteem, and hence to more fulfilling engagement with adult life. They see their job as transformative, rather than as nurturing, so are not afraid to challenge even learners with low self-esteem to work on hard mathematics, make mathematical choices and take risks in a supportive, collegial environment.

For me, this is the central challenge for mathematics teaching in our time - not to underestimate what learners can do. I see this as not just an educational imperative but also one which arises from a social justice agenda. In England, students in our ‘bottom sets’ come overwhelmingly from social groups which are already disadvantaged. For me, the only ethical purpose for separating out those learners who fail to meet expected levels of mathematical knowledge by certain ages should be to give them specially-focused teaching which restores them back to normal levels of functioning in school maths, thus restoring their life chances. If separation does not do that, but merely exacerbates difference, then I believe it to be socially unjust.

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