Dance and mathematics: Engaging senses in learning

Anne Watson

University of Oxford, UK

<anne.watson@edstud.ox.ac.uk>

In this paper I illustrate how kinaesthetic experiences associated with dance might be used in teaching to promote engagement and learning in spatial, rhythmic, structural and symbolic aspects of mathematics.

Introduction

Educational institutions searching for 'quick fix' solutions to underachievement may be tempted to adopt one of the many 'theories' currently offered that advise teaching different students in different ways according to their 'preferred learning styles'. For example, in some schools students are tested to find out if they are visual, aural or kinaesthetic learners and then teachers are advised to teach them accordingly. Gardner (1993), who saw his list of learning styles as a conjectural categorisation, not as a recipe, included 'musical' and 'logico-mathematical' as possible preferences too. As well as the obvious problems in organising radically different treatments in whole class settings, or the social dangers of segregating students identified as 'different', teachers may also be puzzled about how to teach mathematics in kinaesthetic or musical ways, when 'logico-mathematical' seems the obvious way to proceed.

In this theoretical paper I explore how dance, an obvious kinaesthetic activity, has been exploited by some teachers to support the learning of curriculum mathematics (Tytherleigh & Watson, 1987). I take 'dance' to be deliberate, planned movement for aesthetic purpose and/or to express or convey meaning. My data comes from personal experience of working with school students, workshops with teachers, and the reported experiences of teachers on the tasks described below.

There are at least four aspects of mathematics that can be related to dance: spatial exploration, rhythm, structure, and symbolisation.

Spatial exploration

A child's first experience of space is three-dimensional and this is explored through movement both inside (e.g., being in the womb; hiding under a table) and outside (e.g., playing with cubes). We know that most people can learn about shape by physically interacting with it (Tahta, 1999). Indeed some psychologists claim that we learn emotionally and conceptually by interacting with the spatial environment (Winnicott, 1955; Maturana & Varela, 1988; Vygotsky, 1994). Crawling around to explore the environment independently seems to be an essential part of our emotional and social learning. Winnicott describes how small children extend their distance from a caring parent as an expression of their growing independence of thought and self. Maturana and Varela suggest that our physical relationship with the world is all we have from which to develop any kind of 'knowing'. Hence the way that we know things is inextricably connected to our interactions with the physical world. Vygotsky, recognising the overwhelming impact of such interactions, focuses on the ways in which language interacts with experience to create thought and knowledge, a connection that is obvious in our speech (Lakoff & Johnson, 1980, p. 29). Thus, we talk about 'taking an idea forward', 'underlying principles', 'higher order thinking' or 'circular, or direct, lines of argument'.

Believing movement to be a fundamental medium for child development, Laban created an educational form of dance, using interweaved icosahedra to represent directions and qualities of movement, which could be used for different physical and emotional purposes. One icosahedron is used to classify the space around us, while another is used to express the dynamics of dance. He recognised that three-dimensional geometrical forms provided a way to organise and represent space (Laban, 1966). Trisha Brown, a choreographer, used this explicitly in asking dancers to imagine themselves inside a cube and touching various features of it, vertices, midpoints and so on, with various limbs, knees, elbows and so on (Kino, 1999). This strategy of using physical imagination to explore shapes from the inside can be used for geometrical education with students; one could prompt exploration of the curvature of an imaginary sphere, or trace out the arcs of an octahedron, or get a feel for axes and planes of symmetry. Beyond geometry, one could experience the continuity and differentiability of surfaces. It is likely that some students already use imaginary kinaesthetic experiences of this kind in their learning, but being explicit about it makes it available to others too.

Bruner's theory of instruction offers shifts between enacted, iconic and symbolic modes of representation as necessary for conceptual learning (Bruner, 1966). Thus teachers might offer: a practical (kinaesthetic) activity; followed by a visualisation of the activity, such as a diagram or model (the iconic phase); followed by symbols or technical words that encapsulate what has been understood. An example of this in elementary mathematics is the physical counting of objects, followed by a tally (which is also a physical enumeration!), followed by the use of a number symbol. An example from higher mathematics could be the arrangement of tessellating tiles, followed by a diagram of such a tessellation, followed by an encoding of the tessellation which informs a learner about the properties of the pattern, such as what angles meet at the vertices, but does not give a picture of it.

Interpretations of Bruner's idea in practice usually offer enactive, iconic and symbolic modes consecutively, yet researchers in Hawaii have found that young learners respond well to experiences in which these occur simultaneously (Dougherty, 2003). Pictures, symbols, words and actions might be given all at once, and the learner have multiple sensory experiences of a new concept. Neuroscientists confirm the intuitively obvious fact that learners who have been offered several modes of representation generate more brain activity as they try to reconcile these stimuli. A child who traces out jumps along an iconic number-line with a finger, while saying number words, is an obvious illustration of this. Another illustration would be the way in which some teachers simultaneously write algebraic expressions, say what they mean, symbol by symbol, and indicate how the position of terms implies their meaning.

These combinations of experience tend to be used less in higher mathematics, yet there are assumptions about physical relationships in many of the ways we represent mathematics. If we see a geometric diagram as merely horizontal it would be hard to describe some of its properties to someone else but, because we can also allow it to be vertical, we can use words like 'top', 'height', 'bottom', 'above', 'drop' and so on. There are also ways in which assumptions about position might raise questions for learners, for example: does a 'higher power' always result in a 'higher graph'? or when seeing a horizontal numberline as emanating from zero, are numbers 8 and -8 'the same size'? Position and space are intimately bound up in the language in mathematics and our expectations about concepts.

Papert, who invented Logo, understood the importance of connecting action, motion and logical experience (Papert, 1980). If we act out what we think, then the mind has richer information from which to make connections with previous experience, and from which to develop memory, and a deeper connection with the event. Any part of the body can trace out a circle and feel its centre, the constancy of its radius, and the plane in which it lies. That sense of circularity has been used by many teachers to convince learners of the sum of external angles of any polygon. If you walk once along all the edges of an imaginary polygon on the floor, being aware of turning the corners, you know that you have turned through 360 degrees, and you also know where the external angles are and what they look like. In the 'walking round a polygon' task there is a two-way link that teachers exploit between actual physical (enacted) movement, and representations (icons) that are memorable and meaningful. Thus, a connection between action, memory and diagram is created. A diagram is not just a representation of an abstract object; for the students it feels like a map of their own movement.

Papert's ideas are expressed in turtle geometry, which links kinaesthetic, visual and symbolic senses to logico-mathematical senses. Using turtle geometry at its most fundamental level involves instructing a cursor to move in a particular direction for a particular distance, and students who can imagine themselves acting this out have less difficulty choosing an appropriate angle than those who work only from a formal understanding of geometry. Indeed, it is hard to view the senses as actually separate in this kind of activity, just as it is hard for the physical movements of a conductor to be seen as separate from the orchestra, the sound, and the visual symbols of the music.

There can be more to this than 'just' acting out and then doing some formal mathematics as a separate activity. For instance, in dance, the priority is to consider what can be done with the body, not with the mathematics. The drawing of a circle on the floor can be done either through motion of a whole body, or through motion of one part of a body, or through the static imagination, or by imagining the movement of someone or something else, or by actually drawing it so it can be seen visually, or by representing it on a diagram, or by projecting it from somewhere else. For example, the challenge to draw a circle on the floor with your left foot as the centre is fairly easy, but to have your elbow at the centre requires some thought about rotating bodies. Drawing a circle on the floor in the distance with an imaginary laser light shining from your finger is harder and non-trivial.

These activities combine action, imagery and thought in complex ways. There is not simply a unidirectional flow from action to thought, nor from thought to action, but a messy cycling between modes. Sometimes the necessities of action lead to realisation and reasoning ('It must be so because…') and at other times thought has to guide action ('I have to do this because…'). These flows are hard to separate in the nexus of solving the current problem, and indeed may not be separable.

When students engage with such tasks, the role of speech is also multidimensional. A very common phenomenon is for a student to find an individual space, explore the task physically, mutter to herself or himself as if words capture ideas, or as if words provide an alternative form of reasoning that has to be uttered to separate it from physical reasoning, and later discuss their 'results' with others using physical movement to illustrate the words. Thus the balance between physical and verbal forms of reasoning and expression varies at different stages of the work.

These kinds of activity suggest not only that physical exploration of shape gives valuable learning experience, but also that appealing to physicality, that is physical imagery and physical memory, is a useful teaching device. All a teacher is doing when introducing movement to explore shape, or number, or graphs, is making overt use of what is already around in the way students experience the world.

Rhythm

Kinaesthetic and musical sensitivities join together in the rhythms of dance. Many people need to respond physically to certain rhythms, either feeling them resonate within or by toe-tapping or getting up and dancing. How can this be seen mathematically? At a very elementary level, there are links that can be made between rote learning and rhythm, such as choreographing the times tables. Rather less obviously teachers can exploit classical rhythms to develop a sense of fractions, as musical notation does in time signatures and note values. The fraction family of: 1/2, 1/4, 1/8, 1/16, can be added and subtracted in common time[1]; the family: 1/2, 1/3, 1/6, and so on can be manipulated in 6/8 time[2]. The added feature of dance can be used to show students that they know these relationships already through their movement, through their beating out of rhythms, so that fractions express what their bodies can already do.

Use of rhythm has connections with the experience of chanting in mathematics lessons. Once one is 'caught up' in the rhythm one pays attention to pattern rather than meaning, and hence is 'living' in the structure; when the rhythm breaks down one rapidly returns to conscious thought about meaning. An exercise that demonstrates this is to try and chant the successive results of subtracting 1 1/10 (one and one-tenth) starting with 101. Typically, chanters find that their attention varies between individual digits, pattern, fraction subtraction, and what other people are doing (especially when they get lost!).

Of course, chanting can be used in ways that engage the memory as well and the experience is not always useful or helpful for learning! Who has not at some time chanted 'times the top by the top and the bottom by the bottom', maybe without any understanding of what it means or when to do it?

Structure

Abstract representations of structure, such as permutations, combinations, graph theory and groups, are manifested in many traditional dances. English and Celtic country dances and North American square dances, for example, usually include actions of combination and their inverses; provide patterns of interaction that ensure that every possible link in a digraph is exploited; and end up back in a starting position after everyone has had an equal role (Playford, 1651). Typical movements in such dances include: two dancers who join right hands and swing round in a full-circle may then have to join left hands and swing in the reverse direction; the couple at the end of a line of couples may have to do some special movements and then go to the other end of the line, followed by the next couple doing the same thing, and so on until everyone has had a turn. Thus they show the characteristics of a mathematical group (in the abstract algebra sense) and provide a cultural manifestation of a structure that can be expressed through symbols. The experience of identifying what is essentially common about a set of traditional dances, claiming this 'truth' to be a definition, using the definition to create a new dance and finding a way to represent it symbolically, could be called an experience of mathematising. In particular, the need to distinguish between

1. 'common time' means four beats in a bar of music, so one bar can be split easily into halves, quarters, eighths and so on.
2. '6/8 time' means that there are two main beats in a bar, each of which has three little parts, with a rhythm similar to that achieved by saying 'taa-ti-ti taa-ti-ti', thus halves and sixths are easily related. To get thirds one may need to look at 3/4 time which is three main beats in a bar; in this time signature, sixths are half of each beat, whereas in 6/8 they are thirds of each beat. Half of a third is the same as a third of a half!

the dancer, the actions and the relationships between actions replicates the distinctions that have to be made in abstract algebra, yet personal involvement in the action can make these distinctions obvious, whereas students of abstract algebra typically find them quite hard to grasp. European bell-ringing, in which sets of bells are rung in sequences that are permuted gradually until the original sequence reappears, also illustrate these characteristics, as does traditional braid-making from many parts of the world (Roaf & White, 2003). Useful websites about English country dancing and time signatures include:

- http://www.cam.ac.uk/societies/round/dances/elements.htm
- http://datadragon.com/education/reading/timesig.shtml.

Symbolisation

In each of the above examples of the potential use of dance to motivate mathematical ideas I have written mainly about a relationship between movement, mind and memory that develops through experience and the senses. However, Laban used the icosahedron not just to model space, but also to record movement in a symbolic notation. Dancers and choreographers seek notations that can convey the complexities of dance, so that its communication and preservation do not depend on a continuous line of teachers. There are many routes by which notation has been sought. In square dancing, once floor shape is established, movement can be conveyed by words, some of which have to be translated, such as 'dozy-doh' for back-to-back and 'strip-the-willow' for something really complicated. Ballroom dancing can be conveyed by a floor diagram showing foot positions with added nuances to indicate movement type. The 18th century dancemasters had floor diagrams that included signs like musical notes to represent rhythm and movement type (Feuillet, 1700; Tomlinson, 1735; Tufte, 1990, pp. 114–119). These combine iconic and symbolic modes of representation, in that the drawn symbols consist of a scale diagram of positions, with added squiggles that have to be translated to have meaning. In Laban's notation, an abstract level of symbolisation is used and there is nothing left of the icosahedron itself to be seen in the notation — it has to be translated. Thus dance can be used to demonstrate how symbolisation arises from a need to record the seemingly unrecordable. Once symbols exist they can be used to communicate to others who hold the same codes, and also to create new objects through the internal rules of manipulation for those symbols. In this case the new objects are dances, but there is of course a similarity with music here (Fauvel, Flood & Wilson, 2003).

In school, learners can create movements, invent ways to encode the movements, and use these codes to convey knowledge to others. Through trying to do this they can develop an understanding of the need for symbolisation and how it can encapsulate complex situations — a mathematical understanding.

Learning mathematics in dance contexts

In all the above suggestions, what is offered are *not* 'kinaesthetic ways to learn mathematics' if this is taken to mean that mathematics can be learned through actions alone, or that there are some students who can only learn through physical action. Instead, they are examples of how the human mind is able to operate in physical–mental, concrete–abstract and iconic–symbolic modes simultaneously, not cyclically, if the context of activity itself demands engagement of mind and body. Each of these ideas has been used by teachers who recognise that the complexities of dance (or music, or crafts) can be made explicit and used to show firstly the power of mathematics and secondly the powers of the learners themselves to handle such situations.

However, such activities can also be a distraction from mathematics if they are not integrated into the learners' overall mathematical experience. At worst, students only remember the dancing. In order for such experiences to have more than novelty and motivational value, they need to relate closely to classrooms in terms of what it means to do mathematics. Students need to be able to pick up and use some of the ideas presented in novel contexts in their normal lessons, as habit. Innovation, at its best, makes a difference to how learners think in all their experiences of mathematics, not just in the innovative context.

I am not suggesting that dance holds the key to this, indeed it would be very easy for all the potential offered above to dissipate by emphasising only cooperation, rehearsal, product and performance (much as many mathematics lessons do!). Rather, I am offering dance as a mode of revealing and enhancing learners' abilities to do some of the things that are useful in mathematics, and that might lead to a deeper understanding of how to engage in learning.

References

Bruner, J. (1966). *Toward a Theory of Instruction*. Cambridge, Mass.: Belknap Press.

Dougherty, B. (2003) Voyaging from theory to practice in learning: measure up. In B. Dougherty, N. Pateman & J. Zilliox (Eds), *Proceedings of the 2003 Joint Meeting of PME and PME-NA*. Honolulu: University of Hawaii.

Fauvel, J., Flood, R. & Wilson, R. (Eds) (2003). *Music and Mathematics: From Pythagoras to Fractals*. Oxford: Oxford University Press.

Feuillet, R. (1700). *Choreographie, ou l'art de decrier la danse*. Paris.

Gardner, H. (1993). *Multiple Intelligences — The Theory in Practice*. New York: Basic Books.

Kino, C. (1999). Trisha Brown in the drawing room. *Art in America*, April.

Laban, R. (1966) *Choreutics*. London: Macdonald and Evans.

Lakoff, G. & Johnson, M. (1980). *Metaphors We Live By*. Chicago: University of Chicago Press.

Maturana, H. & Varela, F. (1988). *The Tree of Knowledge: The Biological Roots of Human Understanding*. Boston: Shambala Press.

Papert, S. (1980). *Mindstorms: Children, Computers and Powerful Ideas*. Brighton: Harvester.

Playford, J. (1651). *The English Dancing Master*. London: Thomas Harper.

Roaf, D. & White, A. (2003). Ringing the Changes: Bells and mathematics. In J. Fauvel, R. Flood & R. Wilson (Eds), *Music and Mathematics: From Pythagoras to Fractals* (pp. 113–130). Oxford: Oxford University Press.

Tahta, D. (1989). Is there a geometric imperative? *Mathematics Teaching, 129*, 20–29.

Tomlinson, K. (1735). *The Art of Dancing Explained by Reading and Figures*. London.

Tufte, E. (1990) Envisioning information. Cheshire, Conn.: Graphics Press.

Tytherleigh, B. & Watson, A. (1987). Mathematics and dance. *Mathematics Teaching, 121*, 39–43.

Vygotsky, L. (1935). The problem of the environment. In R. van der Veer & J. Valsiner (Eds) (1994), *The Vygotsky Reader* (pp. 338–354). Oxford: Blackwell.

Winnicott, D. W. (1964). *The Child, the Family and the Outside World*. Harmondsworth: Penguin.