**Culture and complexity: framing students’ mathematical experience**

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**Introduction**

In this paper I first explore the history of ideas about culture, and relate those to mathematics. This creates problems and possibilities for the task of connecting what happens in mathematics lessons to students' existing, developing and proximal cultural background.

**What is culture in relation to school mathematics?**

Until Matthew Arnold wrote *Culture and anarchy* in the years following 1869, culture was generally taken to mean inherited knowledge based on ancient Greek and Latin literature and religious structures, and the arts as emanating from these, such as church music. Arnold restated culture to mean the active seeking of perfection, through 'the best' of what is said and done. Identification of 'the best' was informed by a spiritual sense of wonder, awe, beauty, 'sweetness and light', a sense not fully defined except in terms of human response and feelings. Everything else is what he called 'machinery' (Arnold, 1869).

A few years after that, Tylor, an anthropologist, defined culture as "that complex whole which includes knowledge, belief, art, morals, law, custom, and any other capabilities and habits acquired by man as a member of society" (Tylor 1871:1). You can see a shift from ideas given by elite sources, to an appreciation of ideas and experiences that are available to everyone as a human response to a coalescence of human-created societal norms. This shift was pushed further by the anthropologist Boas who split everyday culture available to people acting together in groups, what Arnold would call 'the machinery', away from a rarified, imposed, 'high culture' available through Eurocentric, historically based, class and education (Boas, 1930).

Soon after that, Snow famously split culture not across social lines but across disciplines (Snow, 1959). He more or less ignored the everyday habits and experiences studied by anthropologists and instead talked about the knowledge and values of educated people, arguing for interdisciplinarity, rather than democratisation. We are left therefore with two splits, one between arts and sciences and the other between received elite culture and the anthropological cultures of groups.

In mathematics education, Bishop (1988) took the latter split as worth working on, and in doing so also addressed the former, since he saw mathematics 'culture' as embedded in the ways in which different groups of people engaged in everyday spontaneous, craft-skilled and planned practices of counting, locating, measuring, designing, playing, explaining. I claim that the generalisations and abstractions that arise from these activities are not necessarily mathematical ideas; they are situated in their social purposes such as trading, making, competing, decorating and so on, just as the cultural generalisations of school mathematics (such as 'turn it over and multiply') are also not mathematics. A curriculum focused on everyday cultural manifestations of mathematics will not empower learners to be full players in democracy. Gramsci, the Italian political theorist, recognised this when he claimed that school students should learn Latin for no other purpose than they could achieve alongside the elite, who exercised the cultural hegemony, and communicate with the elite, and hence be more likely to get into powerful roles to enact their own cultural domination (see, for example, 1971).

This leaves us with the problem of knowing what is of universal value in elite culture, and what can be jettisoned. On the way to these empowerment goals an educator could hope that the some students would begin to enjoy the so-called elite culture for its own sake , and maybe even observe ways to transform it for the greater good. As examples, being good mathematics has for several generations been seen as a route out of working class culture for clever students whose arts and language skills might be more restricted by their background; classical drama can be a vehicle for actors from working-class backgrounds to convey serious social messages through engaging with received culture ( e.g. Maxine Peake's recent Hamlet in which she dealt with hypocrisy).

While Bishop's insights are helpful in challenging assumptions about intuitive concepts and cultural support for mathematics learning, a curriculum based on these can trap students into 'the machinery' of life. It works on the emotions of social belonging, and hence fear of alienation: "if you make me do them another way I am disorientated and also alienated from my group"; "I don't want to look like a swot in front of my cool friends". Clashes between school and home are frequent, but these are not necessarily because school is offering high mathematical culture pre-Tylor. Such clashes more usually occur because schools try to offer something different to the high mathematics culture that parents expect schools to offer. Significant numbers of parents expect children to be taught certain arithmetical algorithms, whether they bear any relation to home culture, everyday practices, or even conceptual understanding. A typical Hindu parent of a child in an East End school does not expect their children to be taught Vedic arithmetic methods instead of the high status international methods recognised in examinations, although Vedic arithmetic can be used imaginatively by teachers to introduce several standard algebraic structures. As Snow would ask, who are today's Luddites in discussions about mathematics and background culture? Parents expect schools to enculturate students into school mathematics culture which is neither mathematical nor everyday but has its own sets of values and standards, warrants for truth and purposes which are both a degradation of mathematics and a degradation of everyday culture - it is the machinery, to use Arnold's term, used to get grades (see Watson 2008 for more on this).

Connecting with the everyday culture of adolescents is a difficult task. The whole point about youth culture is that it is dynamic, fluctuating and immediate, You have to give yourself primarily to the task of living it to understand it, i.e. you have to be young and in touch. Importing capabilities and habits from current 'youth culture' into the classroom without a full understanding is doomed; as soon as these ideas enter the classroom they become something else, something schooled and either alien or risible. Instead teachers need to aim to engage students as the people they are by drawing on capabilities as human beings, not by trying to match what happens out of school and getting it wrong. Mathematics educators working with street children in Durban, Natal, realised that adolescent girls *wanted* school to be different and safe, and not a mirror of their out-of-school economic behaviour (Vithal, 2003). With high percentages of children in England living in poverty, similar issues may arise here.

How, then, can we view the challenge to make school mathematics more culturally relevant for all while not degrading or inappropriately assuming either mathematical culture, or any other culture, including everyday culture?

Fisher, Harvard professor of aesthetics, gets somewhere close when he connects wonder with wondering (1999). Instead of allowing 'wonder' to stick around with 'awe' and assuming that everyone reacts similarly to the visual revelations that the inner circle of knowers see as beautiful, he attached it instead to the arousal of curiosity, triggered by something that strikes us as beautiful, puzzling, or extraordinary in some way. One cannot wonder, he claims, about anything that is ordinary. Ordinariness promotes no curiosity, no wondering, no wonder. However, he dissociates himself from what Kant called 'the sublime' since that implies external giving of what is amazing which has a religious edge (Kant, 1790). Instead, wondering arises from something innate within each of us, and has an intellectual edge. In some ways, this dissociation from the sublime is a pity because Kant addresses a 'mathematical sublime' and a ' dynamic sublime'. The former occurs when something "overwhelms imagination's capacity to comprehend it" so we try to reason. A dynamic sublime experience gives the feeling of reason's superiority to nature. We have to submit to nature. Fisher thinks that both of these can generate fear as the motivating emotion, fear of confusion and fear of danger. It is worth remembering this when we try to inculcate interest in mathematical ideas by showing students something we ourselves find wonderful, maybe because we understand it, when our students might feel over challenged and worried that they might not see as much in it as we expect, or as other students might do. So while a sublime experience might evoke fear, powerlessness, or religiosity, a wonderful experience might evoke wonder, intellectual engagement, curiosity and empowerment.

What Fisher's take on wonder and wondering does is focus us on the intellectual capabilities of students rather than the intellectual beauty that our subject is traditionally thought to have. The challenge of connecting cultures in our teaching becomes not one of dressing up the curriculum to attract those interested in Snow's other culture, nor dressing up mathematics to make it look like youth culture, nor even pretending that school mathematics is the same kind of stuff as disciplinary mathematics or even everyday uses of mathematics. Further, the challenge is also to avoid students believing that everyday forms of thought (approximation, inductive reasoning, salience, case specific reasoning and so on) are 'mathematical thinking'. Culture, Arnold says, is "endeavour to combat reason and the will of God by means of reading, observing and thinking" and hence will not be good enough for mathematics, which is not about observable phenomena. Nor are the cultures of weaving, furniture making, disco-planning or making art objects necessarily going to give us mathematics. Tasks have to prompt mathematical *reasoning*, through wondering.

All classrooms exist within school culture, and an overarching set of school mathematics cultural expectations that are not necessarily consonant with mathematics culture. These tasks all recognise that school mathematics culture is a warped version of mathematics. In school there is often a strong focus on answers and generalisations rather than structure; teachers avoid uncertainty; and imagination is seldom relied on. Tasks have pedagogic purpose rather than intellectual purpose; characteristic features of mathematics exploration are limited by time slots and curricular pressures; warrants for truth tend to be correctness, accuracy, and curricular fidelity rather than reason (Watson 2008). The deep task for teachers is not about bringing outside cultures into the classroom, but prompting reasoning, through wondering, in the limited culture of classrooms.

I am going to present four mathematical tasks which have connections severally with: 'high culture'; Snow's two cultures; Fisher's notion of wondering; and popular culture. I want to demonstrate how such cultures can be drawn on to move students into a mathematical culture, via a classroom culture in which the normal nature of school mathematics also exists. In doing so, I am also saying that it is the teacher's responsibility, the teacher's job, to make wondering happen by not being ordinary and not offering ordinary tasks. If tasks are ordinary, all the responsibility is on the doer to carry it out compliantly even if emotion is not engaged. There is no wondering.

**Raphael, Pythagoras and Plato: classical culture**

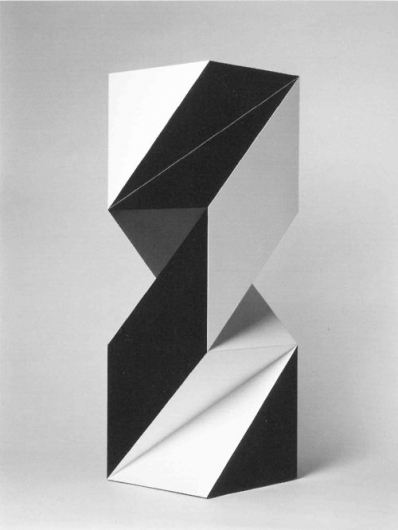
In Raphael's mural 'School of Athens' Euclid is shown with a geometrical diagram on a slate:

While visiting this mural, rather than listening to the guide (who was not speaking my language) my eye was caught by this diagram and the right-angled triangle inside the central hexagon. I thought that Raphael would have known enough geometry to have chosen this diagram with care, and so began my wondering. I noticed that the diagram showed me equilateral triangles on two sides of this triangle, and therefore, using Pythagoras' theorem, I could calculate the size of a triangle on the third side. I mentally completed this new triangle using the dotted lines.

To my surprise and pleasure, the internal triangle whose area I had calculated (dark in figure 2) turned out to be an interesting fraction of the size of the equilateral triangle in which it is contained (pale grey in figure 2) and this gave a moment of wonder. I have since written about this argument in a parody of Plato's record of Meno's discussion with 'the slave boy' (Hamilton and Cairns, 1961; Mason and Watson, 2009). I have been quite brief about this exploration because I do not want to give a full argument and spoil the curiosity of readers. It illustrates a relationship between mathematical exploration and so-called 'high' or classical culture in several ways: classical art uses classical mathematics in covert ways through symmetry and perspective, as well as sometimes using science and mathematics as its subject matter; I used a well-known 'high' culture theorem, that known as Pythagoras', which was widely known across the ancient world in the middle and far east by thinkers and by craftspeople; I used logical reasoning based on known properties and axioms, i.e. mathematical proof, and I knew enough about geometrical reasoning to think of making a suitable construction. I knew enough about mathematics to recognise the value of my findings, and also knew enough about philosophy to recognise a connection between this and a particular dialogue of Plato. Without the suspicion that led me to explore and the reasoning tools with which to explore, I cannot imagine pursuing these ideas. Without the knowledge to frame my surprise at the result, nothing else would have happened. Without all these features, the situation could be reduced to: 'show or find that the ratio of triangle G to triangle H is ....'. In this latter kind of question, there is no discovery and no surprise. On the other hand, this whole task could instead be framed as a role play, pretending that the questions, concerns, and methods of classical scholars are the rules to be followed. What would Euclid ask? What would Euclid do? (Pennington and Faux, 1999)

**Two cultures**

Connections between mathematics and the arts are not guaranteed to be motivating for students. It is difficult enough to interest some students in inherited arts cultures, so even more difficult to do so if the aim is also to get them to do mathematics. The connection between Raphael and the triangle result is as likely to bore some young people as it is to enthuse others. Attempts to connect maths and arts have to engage emotion, curiosity, physical activity, and provide a low threshold of access, so that many pathways are possible. This works best when the arts are authentically based on mathematical principles. Methods I have used frequently include asking students to build a sculpture that requires some spatial reasoning, for example:



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To rebuild this shape from cardboard requires some informal spatial reasoning about symmetries and orientation, and rapidly students find they need to do precise reasoning: what edges are equal; what angles are equal; where the right angles are. The next phase is deductive reasoning about: the implications of the right angles; how to identify and construct parallelograms. Then looking at the properties of the shapes still to be made, and reasoning about their dimensions and properties. Such a task requires students to engage with all kinds of geometrical reasoning - often called the Van Hiele levels (Burger and Shaughnessy, 1986).

I also use dance as a medium to raise mathematical questions (Watson, 2005; see also Baka, 2014) but explanation takes too much space to give details in this paper. Such an approach can be used to create a need for a formal symbol system, and can also display algebraic structures including symmetry groups.

**Wondering**

Creating situations in which 'wondering' arises naturally within mathematics is a way to engender the mathematical cultures of conjecture and exploration, rather than the school mathematics culture of right answers and truncated exploration. Here is one I learned about from Bob Burn:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  | **Σ to n terms** | **Description** |
| 1 | 1 | 1 | 1 | 1 | ... | n | Tallying individual items |
| 1 | 2 | 3 | 4 | 5 | ... | n(n+1)/2! | Counting, 'totals so far' of the line above |
| 1 | 3 | 6 | 10 | 15 | ... | n(n+1)(n+2)/3! | Triangle numbers, 'total so far' of the line above |
| 1 | 4 | 10 | 20 | 35 | ... | conjecture | Tetrahedral numbers, 'total so far' of the line above |
| 1 | 5 | 15 | ... |  |  | conjecture |  |
| ... |  |  |  |  |  |  |  |

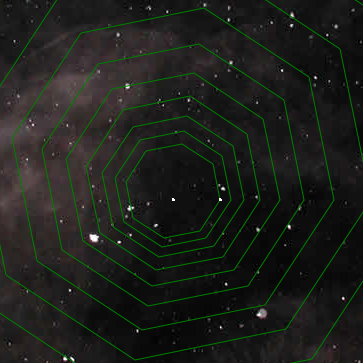
The temptation to extend the pattern in the Σ column is hard to resist, and while students can 'invent' the next formula and test it, it might also be an opportunity to do some necessary algebra, or at least to follow the stages that have to be gone through to find the next formula. Furthermore, if you turn the table through 60ᵒ clockwise you see Pascal's triangle being generated in an unusual way, and students can try and match the reasoning involved in different generating methods.

But as Fisher says, we do not wonder about what is ordinary. If conjectures about patterns, and similar numbers cropping up in unexpected places, is ordinary then how can we expect students to become emotionally engaged in what, for them, might seem to be 'just another investigation'. However, for most of our students the experience of mathematics is more likely to be of many apparently random techniques thrown together which sometimes make patterns and sometimes do not, in which case this approach to Pascal's triangle might be of some interest. The balance between 'ordinary' and 'wonderful' is delicate and cannot be conveyed merely by enthusiasm in the teacher's voice!

**Popular culture**

Combining arts culture with mathematics culture in ways that motivate school students to engage with new mathematical ideas, rather than counting, measuring, estimating and drawing scale diagrams, is a major challenge. If the arts culture you choose is too alien for students the combination will not motivate; on the other hand, if you choose an arts culture that is familiar to students, the mathematical connections may not be authentic. Between these two extremes there is the possibility of combining folk culture with mathematics, such as through the mathematics of weaving, bellringing, country dancing, Rangoli patterns, and so on (e.g. Harris, 1991; Gerdes, 1988). Folk culture, however, is not the shared popular culture of students and may be seen as alien - although the more weird it is the more motivating it can be. Many teachers take a different route and use football as an all-purpose popular culture for frequent contextualisation of mathematics. Recently I was at a mathematics workshop for which the context was Olympic sporting records - my working partner and I did not care enough about any of it to do the mathematics so chatted about something else!

There are other approaches and it is worth searching for them. For example, the website 'teachmathematics.net' has many animations that draw adolescents into needing and doing pure mathematics. In one of these the opening sequence from Doctor Who has been redesigned as an animation using Cabri.



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The task for students is to rebuild it for themselves. To do so they have to engage with the construction of a polygon, and also with its scaling using a geometric sequence including getting the effect of zooming in and out using negative powers.

None of these tasks are ordinary, but 'wonder' is as much due to the learners' disposition as it is of the task itself, and the way it presents surprise and intrigue (MT, 2007).

**Summary and conclusion**

The argument I have been putting forward in this paper is that the relationship between ideas of culture and school mathematics is complex, and simplistic attempts to connect specific cultural aspects of mathematics to elite, folk, interdisciplinary or youth cultures, as perceived by the teacher or the task designer, are likely to fail. Instead I offer a connection between 'wonder' and 'wondering' and suggest that teachers need to do the work to create situations in which it is likely that their students, in their context, with their dispositions, will be caught up emotionally and intellectually in wonder and wondering. These situations must be structurally and challengingly mathematical, rather than situations which can be resolved using everyday, *ad hoc*, inductive or approximate reasoning.

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