Graph-matching situations: Some insights from a cross year survey in the UK

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There are multiple branching curriculum decisions to be made about how functions develop for learners through school. Our main study, from which the data for this paper is taken, contributes by exploring the development of understanding of functions; it compares UK and Israeli students, who learn formally about functions at different ages. This report focuses on covariation – a central aspect – in the “space” of graph-matching situations. Research acknowledges the importance of activities that develop a graphical view of functions as describing variations and for describing situations in problem solving (e.g., Eisenberg, 1991; Swan, 1980). Research also reports on difficulties: the most frequently cited is interpreting a graph as a literal picture of a situation (e.g. Leinhardt et al., 1990). Difficulties with compound variables representing rate and decreasing functions are also reported (ibid.). The aim of this report is to identify some implications of students’ choices in graph-matching situations throughout school in the UK.

We report findings from four graph-matching tasks derived from Swan (1980), and data from 120 UK students, 20 from each of years 7 to 11: 10 from a high-achieving class (A), 10 from a middle-achieving class (B) in each school, and 10 from the first and second years of post-16 mathematics study. Students were asked to match four situations to graphs, write their chosen variables on the axes, and provide explanations for their choices. All situations focused on identifying the variables, forming the relation between them (in particular, capturing their covariation), and noticing contextual features. Variables of different kinds were used, which, as indicated above, may be linked to differences in the quality of responses. Due to lack of space we give just one example: “After the concert there was a stunned silence. Then one person in the audience began to clap. Gradually, those around her joined in and soon everyone was applauding and cheering” (unidimensional variables; increasing function). The full task, including the variety of graphs, appears in Ayalon et al. (2013).

An iterative and comparative process of analysing 480 students’ responses led to three codes: (1) No choice, often accompanied by “I don’t know”; (2) Lack of full analysis; (3) Full analysis. Further analysis of code 2 led to four sub-categories of difficulties:

* Focus on one variable with picture/graph confusion (PG).
* Choosing one correct and one irrelevant variable forming an irrelevant relation (IV).
* Forming an inadequate relation between the variables (IR).
* Failing to notice contextual features (CF).

Analysis of the results of A classes, in all situations, appeared to suggest that there is identifiable age-related progress towards full analytical interpretation. In the early years most responses reflected an inability to approach the task (code 1) and as years progressed responses of full analysis became more apparent (code 3) but the occurrence of code 2 was similar throughout all years. While we expected students to make progress throughout school, and aimed to describe such progress, for the B classes progress was not obvious. We are currently probing many reasons for this including teacher and school variables.

Further analysis of the responses associated with code 2 (194 out of the 480) revealed that categories IV, IR, and CF constituted, each, about 30% of the responses. In contrast, PG constituted only 5% of the responses. These findings point at the criticality of identifying variables and interpreting covariation, rather than the picture/graph confusion prominent in the literature. In total, in both A and B classes, there was little variation in the frequency of categories across years, but category frequency *did* vary between situations: 100% of responses associated with category PG appeared in one situation only, 83% of responses associated with CF appeared in one situation, and about 50% of responses associated with each of the categories IVand FR appeared in each of two specific situations. These findings, though in a limited sample, suggest that certain sources of difficulties may be associated with some situations and not with others, and they persist in advanced years. The variables and context of a situation may significantly shape students’ ability to graph-match. Moreover, although some possible explanations may be suggested, further questions can also be raised about specific characteristics of situations and their relation to success/failure in identifying variables and their relations. Further work is intended to connect to data from other tasks included in our research.

**References**

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