**Adventure and adolescence: learner-generated examples in secondary mathematics**

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In this paper I will show how adventure in mathematical learning relates to the adolescent project of negotiating adulthood. Further, I will relate this to some of the ‘essential learnings’ identified by Tasmanian educational policy-makers by illustrating how autonomous thinking might be embedded in the teaching and learning of mathematics.

This kind of comment is usually taken to mean that mathematical tasks need to be ‘relevant’. Then the word ‘relevant’ is often taken to mean that adolescent interests, (such as music, skateboarding and pocket money), should provide contexts for mathematical tasks – otherwise they will not be interested. The consequence of this line of thought can be that mathematical calculations are dressed up in artificial contexts, yet adolescent students are not fooled by this. A more interesting consequence is that students can be asked to solve ‘real life’ problems in the classroom in the hope that through this process they will learn some mathematics. Thus they calculate batting averages, compare journeys to school, divide *n* dollars between *m* people, but these experiences do not necessarily lead to further knowledge of means, graphs, gradients, or ratio.

We know, thanks to the work of Freudenthal, that we can explain this problem not in terms of poor teaching or lack of learning effort but in terms of the difficulty (Lave would say the impossibility (Lave and Wenger 1991)) of lifting the ways we think in one context and applying them in other contexts (Freudenthal 1973 p.130). When we compare pocket money we are in the world of money and adolescent notions of fairness, not in the abstract world of ratio; when we compare journeys to school we are imagining ourselves and our mates arriving at the school gate, not constructing images of time-distance relationships (Coleman & Hendry 1990 p.48). Shifts from proximal, *ad hoc*, methods of solution to abstract concepts are hard to make and need deliberate support – indeed this is what is at the heart of Vygotsky’s insistence that talk is a necessary aspect of learning ‘scientific’ concepts (1978 p.131), otherwise one gets stuck with intuitive and everyday notions such as ‘multiplication makes things bigger’ or ‘the bigger the perimeter the bigger the area’ (Fischbein 1987). This shift towards seeing abstract patterns and structures within a complex world is typical of adolescent development (Coleman & Hendry 1990 p. 47) but the verbal and kinaesthetic socialised responses to sensory stimuli (including maths questions) which have satisfied teachers at elementary level have to be put to one side.

Realistic tasks - tasks that mimic the ways we think in complex, messy, everyday situations – can provide contexts in which mathematics can be learnt, so long as the ideas encountered while doing the tasks can then be related to overarching mathematical ideas and universal structures through paying attention to patterns, properties and relationships. This is what (Treffers 1987) has called ‘vertical mathematising’. Furthermore, the processes of working with such tasks appeal to adolescents because they provide room for adolescent concerns about identity, belonging, being heard, being in charge, being supported, feeling powerful, understanding the world, and being able to argue in ways which make adults listen (Boaler 1997).

Such tasks can indeed provide contexts for inquiry, as defined in the Tasmanian curriculum document, which includes “identifying and clarifying issues, and gathering, organising, interpreting and transforming information. It encompasses the processes of creatively, imaginatively and inquisitively thinking about possibilities; analysing, synthesising and evaluating proposed solutions; and explaining and justifying decisions. The skills of inquiry can be used to clarify meaning, draw appropriate comparisons and make considered decisions.”

Historically, mathematics has been inspired by observable phenomena, and mathematicians develop new ideas by exploring and inquiring. It is also possible to conjecture relationships from experience with examples, and thus getting to know about general behaviour. But mathematics is not essentially an empirical subject at school level. Its strength and power are in its abstractions, its reasoning, and its hypothesising about objects which only exist in the mathematical imagination. Many secondary school concepts are beyond observable manifestations, and beyond intuition. Indeed, those which cause most difficulty for learners and teachers are those which require rejection of intuitive sense and reconstruction of new concept-based images and understandings. Examples of these problematic topics include probability, proportional relationships, non-linear sequences, symbolic representations and the wretched adding of fractions. In mathematics, inquiry alone cannot fully justify results and relationships, nor can decisions be validated by inquiry alone.

I could argue that, for the adolescent, this can be the beginning of the end of mathematical engagement. If I cannot understand the subject by seeing what it does and where it is and how it works, but instead have to believe some higher abstract authority that I do not understand, then it holds nothing for me. But this misses the point. The authority of mathematics does not reside in teachers or textbook writers but in the ways in which minds work with mathematics itself (Freudenthal 1973 p.147; Vergnaud 1997). For this reason, mathematics, like some of the creative arts, can be an arena in which the adolescent mind can have some control, can validate its own thinking, and can appeal to a constructed, personal, authority. In mathematics, I can always support my thinking by looking at my work in a different way – and adjusting it if necessary or seeking help on my own terms. Thus, in mathematics, there is always the possibility that learners can be absolutely sure they are right, and have grounds to argue with. By ‘sure’ I do not necessarily mean the use of mathematical proof – although if a learner understands a proof this is one way to be sure. Instead I mean that they can back their arguments with demonstration, generalities, counter-examples and other tools of intellectual expertise.

Jenny Houssart, in her research with pre-adolescents, spotted some students who were rapidly becoming disaffected in the classroom, with a hostile atmosphere between them and the teacher. One feature of this deteriorating relationship was that they were sometimes right about maths when the teacher was wrong, but their comments were ignored, sidelined, or even punished by the teacher (2004).

By contrast, students whose ideas and thinking are valued explicitly in lessons are more likely to feel and behave as if they belong. Thus some adolescent boys arrange themselves to be directly in the teacher’s line of vision and make sure she knows that they know the answers, or that they understand what is going on. As the Tasmanian curriculum document says: “People who have a sense of competence in their ability to think and learn … will be eager to pursue questions that really matter.”

Those whose thinking never quite matches what the teacher expects, but who never have the space, support and time to explore why, can become disaffected at worst, and at best come to rely on algorithms. While all mathematics students and mathematicians rely on algorithmic knowledge sometimes, to have that as the only option places learners totally at the mercy of the authority of the teacher, textbooks, websites and examiners for affirmation. Since a large part of the adolescent project is the development of autonomous identity, albeit in relation to other groups, something has to break this tension – and that can be a loss of self-esteem, rejection of the subject, or disruptive behaviour (Coleman & Hendry 1990 pp.70, 155). I would argue that presenting mathematics in imaginary contexts does not necessarily touch their need for self-actualisation; nor do contrived references to what might be useful in future employment. What is required instead are ways to engage the person as they are here and now in the human activity of doing mathematics using their own thinking. This might include contexts of genuine interest, and information about how human beings developed the subject by asking pertinent and curious questions, but it could also include intriguing and puzzling situations, unanswerable questions, ways to use what they already know to generate new big ideas.

My aim is to develop ways in which *all* students, not just those whose arm-waving attracts the teacher’s positive attention, can be engaged in mathematics for its own sake and thus begin to see that mathematical thinking is a part of who they are, and might form a part of who they become. In the development of adolescent identity, I suspect that using football as a context for mathematics affirms the identity of football fans; using the school disco affirms the identity of organisers and music freaks. The ability of the stronger mathematicians to take over such contexts and claim ownership can exclude and deskill and therefore further dishearten some of the students whom the context was supposed to help. In addition, Cooper and Dunne’s finding that context can confuse students from disadvantaged groups because it supplies a layer of necessary discernment about what is, and is not relevant (2000). Instead, there is a mathematical component of identity – the human capacity to reason spatially, numerically and logically, which can be nurtured by participation in mathematics, by having one’s thinking valued, and by having some autonomous control over the locus of mathematical understanding in lessons.

I shall now give some examples of tasks which generate and nurture this aspect of identity, and which fit very well into Tasmania’s description of ‘essential learnings’. Each task is of a type that can be applied to many mathematical contexts. I am not claiming these task-types are new, but representing them as tools for engaging adolescents into personal engagement with mathematics, and ceding authority and validation to their relationship with mathematics.

**Learner-generated examples**

Students in a lesson were familiar with multiplying numbers and binomials by a grid method:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 20 | 7 |  | X | z | +3 |
| 50 | 1000 | 350 |  | 2z | 2z2 | 6z |
| 9 | 180 | 63 |  | -1 | -z | -3 |

They had been introduced to numbers of the form *a ±√b*. The teacher then asked them to choose pairs of values for *a* and *b*, and to use the grid method to multiply such numbers to try to get rational answers. Students worked together and began to explore. At the very least they practised multiplying irrationals of this form. Gradually, students chose to limit their explorations to focus on numbers like 2 and 3 and, by doing so, some realised that they did not need explicit numbers but something more structural which would ‘get rid of’ the roots through multiplication. Although during the lesson none found a way to do this, many carried on with their explorations over the next few days in their own time.

Tasks in which students gain technical practice while choosing their own examples, with the purpose of finding a particular property or relationship, can be adapted to most mathematical topics. During the work there are many non-trivial features of their thinking that are valuable and can be praised: exemplifying, controlling variables, conjecturing, limiting the range of variation used, observing and testing special cases, designing spreadsheets to carry out the task, seeing implications of some results, generalising and so on.

**Equivalence**

Shifting the focus of lessons from finding answers to generating equivalence alters the locus of power in lessons. Here are two examples:

Given a point on a number line drawn on grid paper, how many ways of representing it as a fraction can you find?

This reverses questions which ask people to reduce fractions to their simplest form. It also gives each student a set of fractions they can use for future activity, augmented by each other’s findings – a kit of examples for demonstrating and validating future calculations.

Given *y* = 5, how many equations can you write in which *y* is secretly 5, but this fact is hidden, such as in 17 = 3*y* + 2?

This task generates a set of questions which everyone can then try to answer – the class has created its own practice exercise.

**Another and another …..**

Ask students to give you examples of something they know fairly well, then keep asking for more and more until they are pushing up against the limits of what they know.

Give me a number between zero and a half; and another; and another …

Now give me one which is between zero and the smallest number you have given me; and another; … and another….

Each student works on a personally generated patch, or in a place agreed by a pair or group. Teachers ensure there are available tools to aid the generation – in this case some kind of ‘zooming-in’ software, or mental imagery, would help.

This approach recognises their existing knowledge, and where they draw distinctions; it then offers them opportunity to add more things to their personal example spaces, either because they have to make new examples in response to your prompts, or because they hear each other’s ideas (Watson & Mason 2006). Self esteem comes at first from the number of new examples generated, then from being able to describe them as a generality, and finally from being able to split them into distinct classes.

**Putting exercise in its place**

If getting procedural answers to exercises in textbooks is the focus of students’ mathematical work (whether that was what the teacher intended or not) then shifts can be made to use this as merely the generation of raw material for future reflection. Many adolescents have their mathematical identities tied up with feeling good when they finish such work quickly, neatly and more or less correctly; others reject such work by delaying starting it, working slowly, losing their books and so on. Restructuring their expectations is, however, easy to do if new kinds of goal are explicated which expect reflective engagement, rather than finishing, so that new mathematical identities can develop more in tune with the self-focus of early adolescence (Dweck 1999).

Examples of different ways to use exercises are:

Do as many of these as you need to learn three new things; make up examples to show these three new things

At the end of this exercise you have to show the person next to you, with an example, what you learnt

Before you start, predict the hardest and easiest questions and say why; when you finish, see if your prediction was correct; make up harder ones and easier ones.

When you were doing question N, did you have to think more about: method, negative signs, correct arithmetical facts, or what? Can you make up examples which show that you understand the method without getting tied up with negative signs and arithmetic?

Since the Tasmanian ‘essential learnings’ have been written to cover all subjects, many of the special features of mathematical thinking are only hinted at. ‘Active reflection’ it says ‘enables connections to be made between different types of subject matter, and this enhances the likelihood of knowledge being transferable to new situations’. This is true, of course, but the active reflection I describe here does more: it enables connections to be made *within* particular concepts and methods, so that learners become better at developing critical relational knowledge. It is this recognition of methods at a structural, rather than operational, level that makes adaptable and transformable understanding more likely.

**Rules *versus* tools**

‘Learning is more effective, interesting and relevant when learners consciously choose and use particular methodologies, devise their own strategies to deal with challenges’ says the Tasmanian document.

Student-centred approaches often depend on choice of method, and this, of course, celebrates autonomy. However, mathematics is characterised by, among many other things, variation in the efficiency and relevance of methods. For example, ‘putting a zero on the end when multiplying by ten’ is fine so long as you are working with whole numbers – and mathematicians do not abandon that way of seeing it. Rather than it being a *rule* it becomes a *tool* to be used when appropriate. Adolescents often cannot see why they should abandon methods which have served them well in the past (repeated addition for multiplication; guessing and checking ‘missing numbers’; and so on) to adopt complex algorithms or algebraic manipulations. One way to work on this is to give a range of inputs and to ask students to decide which of their methods works best in which situation, and why. This leads to identifying methods which work in the greatest range of cases, and the hardest cases. ‘Supermethods’ need to be rehearsed so that they are ready to use when necessary, and have the status of tools, rather than rules.

In all of the above task-types, students create input which affects the direction of the lesson and enhances the direction of their own learning. I see this as an adventure, since they are each starting out from the safe ground of their own knowledge-so-far and moving elsewhere within a mathematical community. Classrooms in which these kinds of task are the norm provide recognition and value for the adolescent, a sense of place within a community, and a way to get to new places which can be glimpsed, but can only be experienced with help. To use the ‘zone’ metaphor – these tasks suggest that mathematical development, relevance, experience and conceptual understanding are all proximal zones, and that moves to more complex places can be scaffolded in communities by the way teachers set mathematical tasks.

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